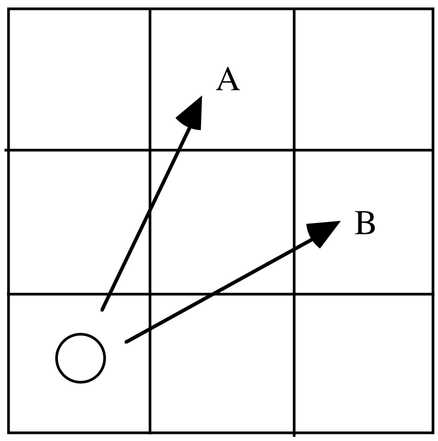
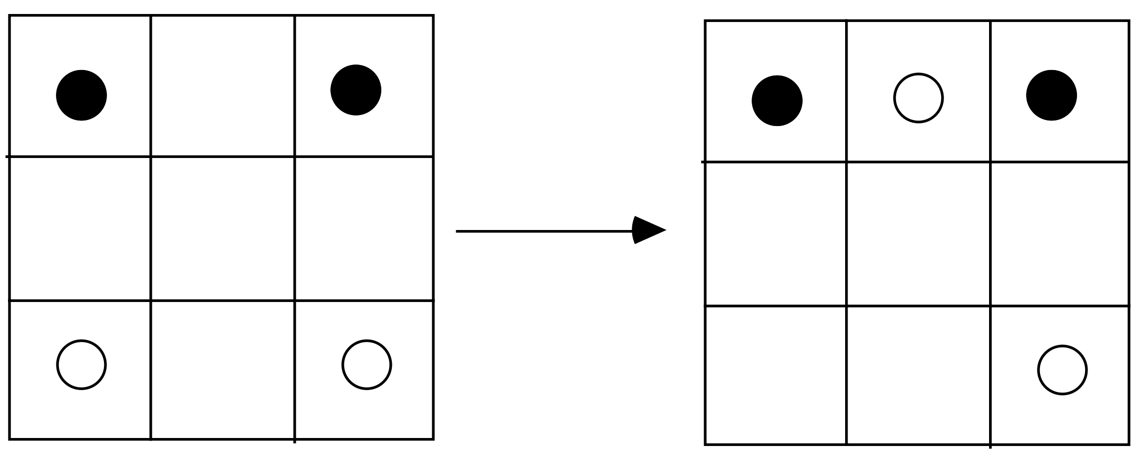
Math 215 - Assignment Number 1, Spring 2013
Hand these problems in on Monday, January 21.

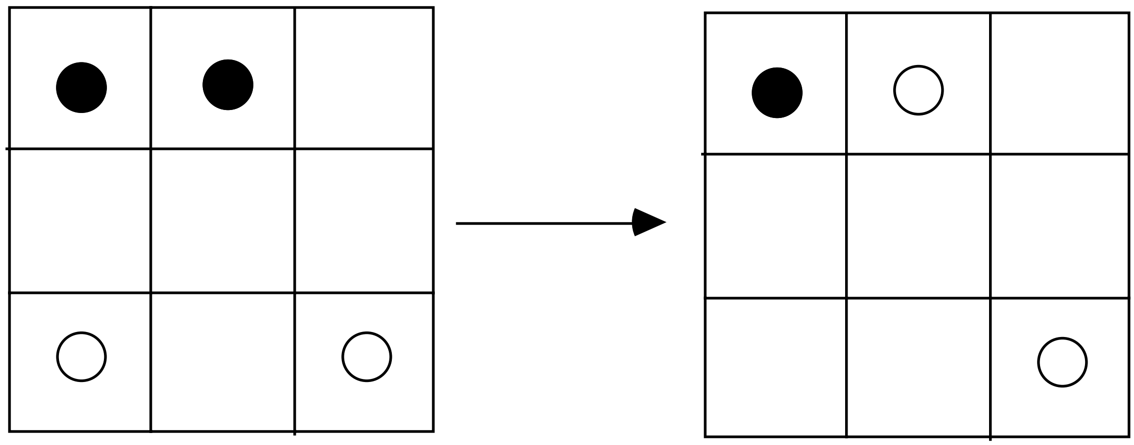
1. Consider the game of chess played only with knights on a 3 x 3 board. We will use black and white dots to indicate the knights. A knight moves by going from its present square to any nearby square that can be reached by moving 2 squares horizontally and 1 square vertically or by moving 2 squares vertically and 1 square horizontally.



In the figure above, the white knight at the lower left can move to either square A or to square B. As long as A or B are unoccupied, then the knight can move to those squares. It is not required that any intermediate squares are occupied. Thus the following is a legal move in a game between Black and White.

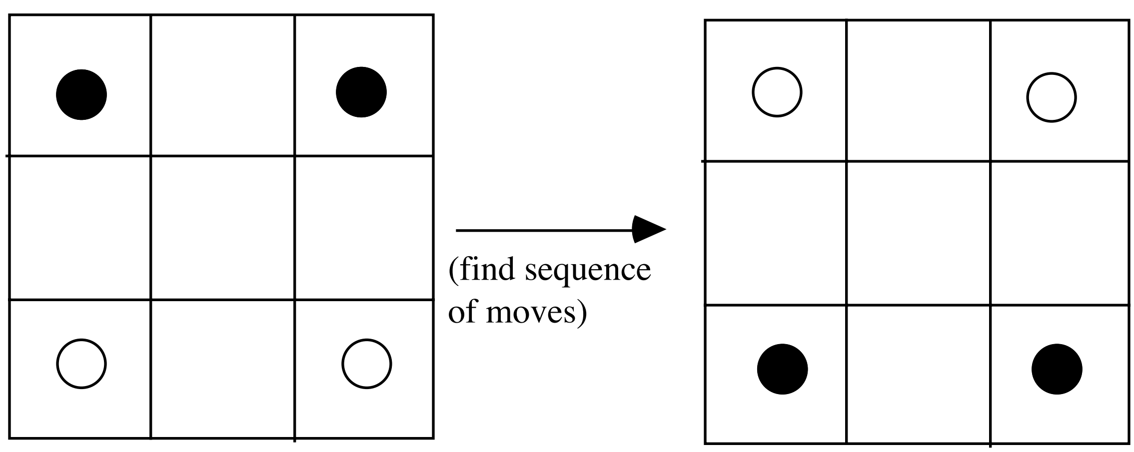


If the end-square of a move is occupied by your opponent, than you can capture his piece, as shown below.



Players alternate turns with first White and then Black moving. In usual games of chess, we designate the possibility of winning by capturing pieces from your opponent. In this problem, you will be asked to find out whether or not there is a sequence of legal moves from one position to another.

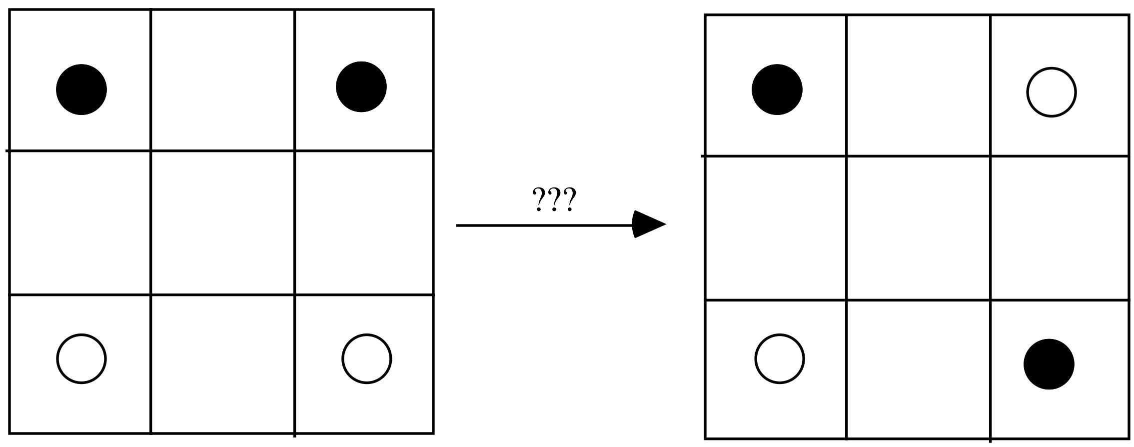
Question A. Show explicitly a sequence of legal moves (White, Black, White, Black, ...) from the first position to the second position.



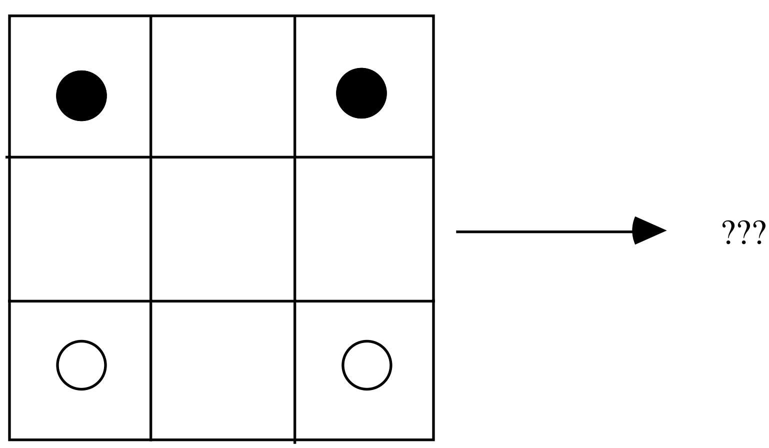
Of course you are not allowed to rotate the board! You have to move the knights alternately: white, then black, then white and so on, and get from the left position to the right position.

Question B. Decide whether you can get from the left position below to the right position below by a sequence of

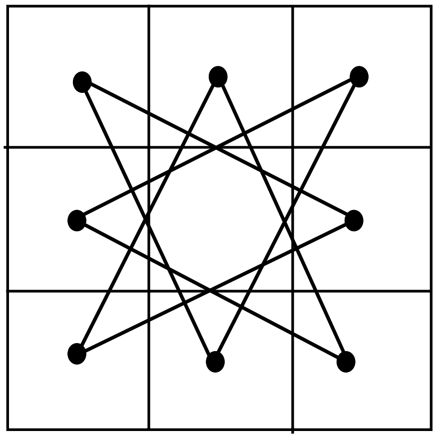
legal moves. How can you be completely sure of your answer if there is no way to do it?



Question C. Can you find all possible positions of the knights that can be reached by legal game moves from the left hand positions of these two problems? How can you know that your answer is complete?



Remark. It will help to consider all the ways that a single knight can move on the board.



2. Read in Eccles Chapters 1,2,3,4,5 (pp. 1-52) and *be responsible for the exercises for Chapters 1 and 2.* (The solutions are in the back of the book, so we are not asking you to hand in these exercises, but you should do them and compare your answers with the book. In class, please feel free to ask questions related to these exercises.)

3. Symbolic logic uses the following notation (in this word processor!):
AvB = A or B,
A^B = A and B,
A > B = A implies B,
~A = not A.
Take X = Y to mean that *X and Y have exactly the same truth tables* when X and Y are expressions in symbolic logic that have truth tables.

(Note that if X and Y have the same truth table and Y and Z have the same truth table, then X and Z also have the same truth table. Thus we have shown that if X = Y and Y = Z then X = Z. By the same token we have that X = X and that if X = Y then Y = X. In other words, the equality we have defined for expressions in symbolic logic has the same basic properties as equality in other parts of mathematics. A notion of equality that satisifies these three properties is called an *equivalence relaton*, and we will have more to say about this concept.)

Prove the following equalities by showing that the two sides have the same truth tables.

(a) $\sim\sim P = P$.
(b) $F \vee P = P$.
(c) $A \vee \sim A = T$.
(d) $\sim(A \wedge B) = (\sim A) \vee (\sim B)$.
(e) Given the truth table for $P > Q$ as on page 11 Table 2.1.1 of Eccles, Show that $(P > Q) = (\sim P) \vee Q$ by comparing the truth tables.
(f) $(P > P) = T$.
(g) $(F > P) = T$ for any P. This says that a false proposition implies anything. For example it is true that "If elephants can fly then turtles can run the three minute mile.".
(h) $(P > \sim P) = \sim P$ for any P.
(i) $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$.
(j) $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$.

4. You can work algebraically with identities in symbolic logic. Suppose you are given that $A \vee \sim A = T$ for any A and that $\sim(P \vee Q) = (\sim P) \wedge (\sim Q)$ and (of course) that $\sim T = F$. Prove, without using truth tables, that $\sim A \wedge A = F$ for all A.

5. Write a proof of the Pythagorean Theorem along the lines of the one we give in class. Organize your proof so that it could be understood easily by a student just beginning to study geometry. Your proof will have to depend on some given mathematical facts. In writing the proof you must state these facts carefully and then use them in your proof.