Math 215 - Assignment Number 5, Spring 2013

Read Chapters 10,11,12. Read the notes on sets by Rudin that we have on our website.

1. Eccles page 115-119. problems 3,5,11,12,13,14,15,16,22.
2. Prove that the number of subsets of the set \{1,2,\cdots,n\} is \(2^n\). You may wish to do this by induction on \(n = 1,2,3,\cdots\).
3. Prove again that the number of subsets of the set \{1,2,\cdots,n\} is \(2^n\), this time by using the binomial theorem for \((x+y)^n\) with \(x = y = 1\).
6. The following is a “proof” that “All horses have the same color.” What is wrong with this proof?

**Theorem.** All horses have the same color.

**Proof.** We will prove this theorem by induction on \(n\) where \(n\) is the number of horses. For \(n = 1\) we have one horse and obviously this horse has one color. (We assume that each horse has a definite color.). Now suppose for the induction hypothesis that *any* \(k\) horses have the same color for some specific natural number \(k\). We wish to show that any \(k+1\) horses have the same color. So let \(k+1\) horses be given and choose one of the horses, call it A, and put it aside. We now have \(k\) horses and so by the induction hypothesis, they all have the same color. Now take the horse A that we put aside and add it to this group, but take away another horse B. Now the horse A belongs to a group of \(k\) horses and so must be the same color as them. But B has this color and so A and B have the same color. We have shown that all the horses except A have the same color and that B (a member of all the horses except A) has the same color as A. Thus all \(k+1\) horses have the same color. We have shown that if every group of \(k\) horses have the same color, then every group of \(k+1\) horses have the same color. This completes the induction proof that all horses have the same color. QED.

7. The following is a “proof” that \(1 = 0\). What is wrong with this proof? Begin with \(x\) and \(y\) non-zero and \(x = y\). Then \(x = y\) implies that \(x^2 = xy\), and subtracting \(y^2\) from both sides, we have \(x^2 - y^2 = xy - y^2\). Now divide both sides by \(x - y\) and get \(x + y = y\). But since \(x = y\) we then have \(2y = y\) and since \(y\) is non-zero we divide by \(y\) and get \(2 = 1\). Subtracting 1 from both sides, we have shown that \(1 = 0\). QED.