These are very sketchy notes indicating just some bare essentials of plane geometry and some problems to think about.

**Axioms**
1. Two points determine a line. That is, if two lines each contain distinct points $p$ and $q$, then the two lines are identical.
2. All straight angles are equal.
3. Given a line $L$ and a point $p$ not on $L$, there is a unique line through $p$ that is parallel to $L$. (Two lines are parallel iff they do not meet.)
4. Given two parallel lines $L_1$ and $L_2$, let $L$ be a line transverse to both $L_1$ and $L_2$ (i.e. $L$ intersects $L_1$ in one point and $L$ intersects $L_2$ in one point as well). Then the corresponding angles between $L$ and $L_1$ and between $L$ and $L_2$ are equal.

\[ \text{Angle } a = \text{Angle } b. \]

**Exercises.**
(a) Prove that vertical angles are equal.

\[ \text{Angle } a = \text{Angle } b \]

(b) Prove that the sum of the angles of a triangle is equal to
a straight angle.

(c) Prove the Pythagorean Theorem: If \( a \) and \( b \) are the lengths of the sides of a right triangle and \( c \) is the length of the hypotenuse (the hypotenuse is the side opposite the right angle. a right angle is one half of a straight angle), then \( a^2 + b^2 = c^2 \).

(d) Observe that \((x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2\).
This formula can be used to produce triples of numbers like \((3,4,5)\) where \(3^2 + 4^2 = 5^2\). These are called Pythagorean Triples. Investigate this phenomenon.

(e) \textbf{The Pentagon and the Golden Ratio}

Contemplate the following figure.

The small pentagon has its edge \(e\) and chord \(d\) labeled. We embed the small pentagon in the larger one and observe via a parallelogram and by similar triangles that \(d/e = (d+e)/d\) (similar triangles).
Thus with \(F = d/e\) we have
\[
d/e = 1 + e/d\]
whence
\[
F = 1 + 1/F.
\]
This is sufficient to show that \(F\) is the golden ratio. Note how the golden ratio appears here through the way that a pentagon embeds in a pentagon, a pentagonal self-reference.

(f) \textbf{The Trefoil Knot, the Pentagon and the Golden Ratio}
Here is an experiment that you can do with the trefoil knot and a strip of paper. Tie the strip into a trefoil and pull it gently tight and fold it so that you obtain a flat knot.

As you pull it tight with care a pentagon will appear!

Now it is intuitively clear that this pentagonal form of the trefoil knot uses the least length of paper for a given width of paper (to make a flattened trefoil). At this writing, I do not have a proof of this statement.