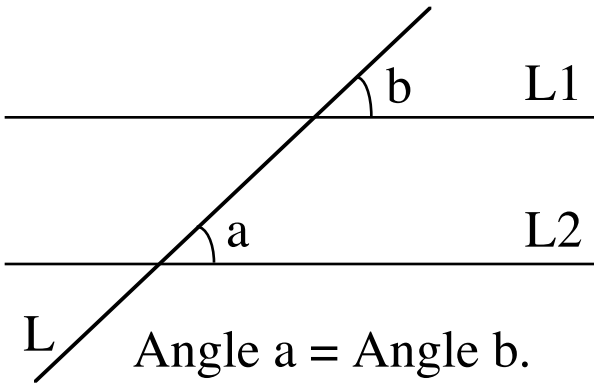
Plane Geometry

by LK

These are very sketchy notes indicating just some bare essentials of plane geometry and some problems to think about.

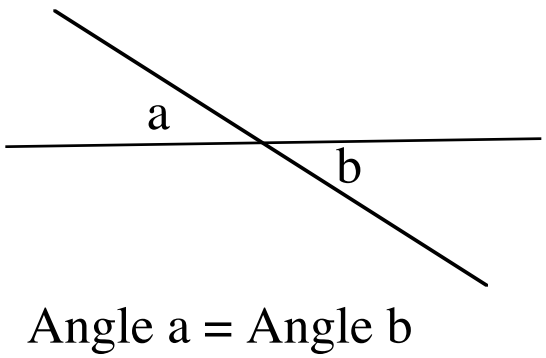
Axioms

1. Two points determine a line. That is, if two lines each contain distinct points p and q, then the two lines are identical.
2. All straight angles are equal.
3. Given a line L and a point p not on L, there is a unique line through p that is parallel to L. (Two lines are parallel iff they do not meet.)
4. Given two parallel lines L1 and L2, let L be a line transverse to both L1 and L2 (i.e. L intersects L1 in one point and L intersects L2 in one point as well). Then the corresponding angles between L and L1 and between L and L2 are equal.



Exercises.

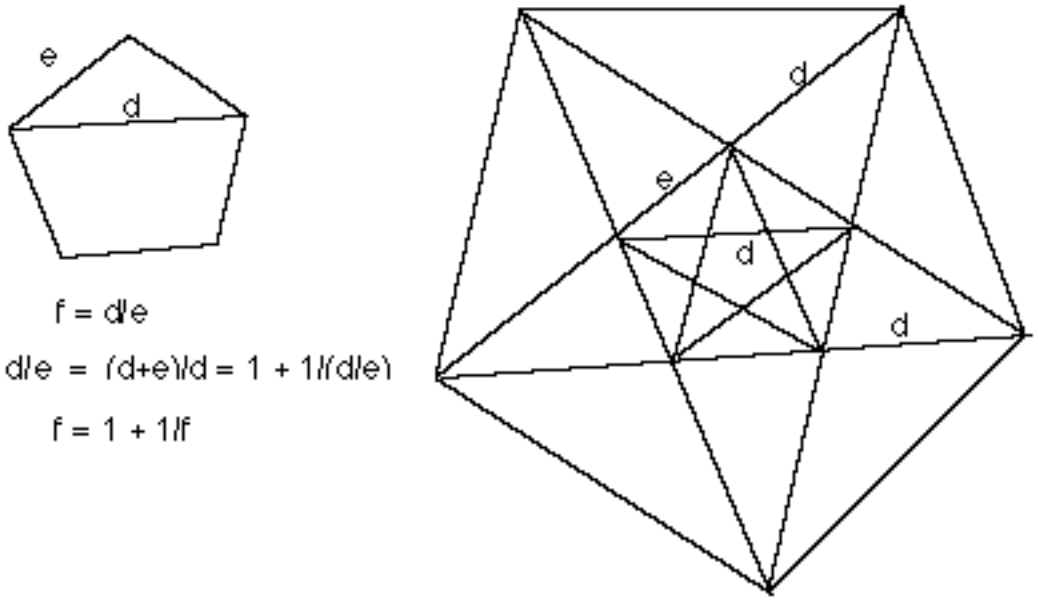
(a) Prove that vertical angles are equal.



(b) Prove that the sum of the angles of a triangle is equal to

a straight angle.
(c) Prove the Pythagorean Theorem: If a and b are the lengths of the sides of a right triangle and c is the length of the hypotenuse (the hypotenuse is the side opposite the right angle. a right angle is one half of a straight angle), then $a^2 + b^2 = c^2$.
(d) Observe that $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$.
This formula can be used to produce triples of numbers like (3,4,5) where $3^2 + 4^2 = 5^2$. These are called Pythagorean Triples. Investigate this phenomenon.

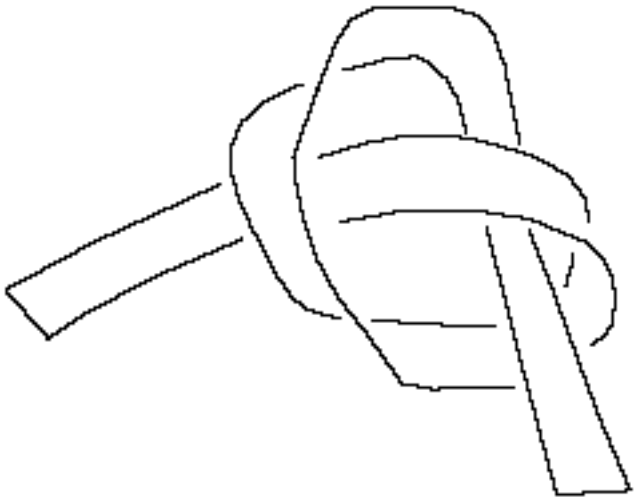
(e) **The Pentagon and the Golden Ratio**
Contemplate the following figure.



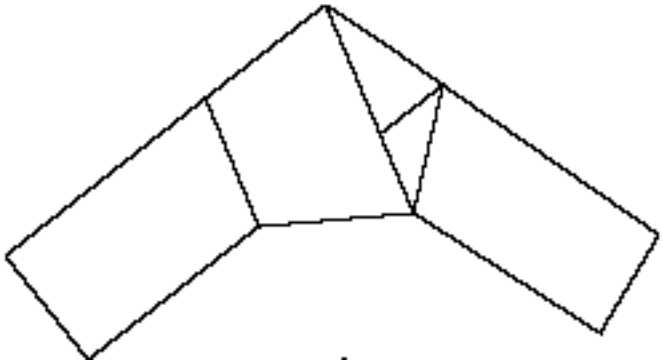
The small pentagon has its edge e and chord d labeled. We embed the small pentagon in the larger one and observe via a parallelogram and by similar triangles that
d/e = (d+e)/d (similar triangles).
Thus with F = d/e we have
d/e = 1 + e/d whence
F = 1 + 1/F.
This is sufficient to show that F is the golden ratio. Note how the golden ratio appears here through the way that a pentagon embeds in a pentagon, a pentagonal self-reference.

(f) **The Trefoil Knot, the Pentagon and the Golden Ratio**

Here is an experiment that you can do with the trefoil knot and a strip of paper. Tie the strip into a trefoil and pull it gently tight and fold it so that you obtain a flat knot.



As you pull it tight with care a pentagon will appear!



Now it is intuitively clear that this pentagonal form of the trefoil knot uses the least length of paper for a given width of paper (to make a flattened trefoil). At this writing, I do not have a proof of this statement.