QuizNumber2, Solutions, Math 210, Spring 2016

1. (a) Given the vectors $\vec{v} =< 1, 2, 3 >$ and $\vec{w} =< 1, 1, 1 >$, find the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$ in three-space.

Ans. The area of the paralleogram spanned by the two vectors is the length of their cross-product

$$|\vec{v} \times \vec{w}| = | -\vec{i} + 2\vec{j} - \vec{k}| = | < -1, 2, -1 > | = \sqrt{6}.$$

(b) For the vectors in part (a), find the vector projection of $\vec{v}$ on $\vec{w}$ and find the length of the projection of $\vec{w}$ on $\vec{v}$.

Ans.

$$P_{\vec{w}}(\vec{v}) = \frac{\vec{v} \bullet \vec{w}}{\vec{w} \bullet \vec{w}}\vec{w} = \frac{6}{3}\vec{w} =< 2, 2, 2 > .$$

$$P_{\vec{v}}(\vec{w}) = \frac{\vec{v} \bullet \vec{w}}{\vec{v} \bullet \vec{v}}\vec{v} = \frac{6}{14}\vec{v} = \frac{3}{7} < 1, 2, 3 > .$$

$$|P_{\vec{v}}(\vec{w})| = |\frac{3}{7} < 1, 2, 3 > | = \frac{3}{7}\sqrt{14}.$$

2. A ball of mass m is initially at the point $(1, 1)$ in the plane and is subject to a gravitational force of $< 0, -mg >$. The ball is given an initial velocity $< cos(\pi/4), sin(\pi/4) >$. Here $g = 32ft/sec^2$, and the velocity components are in ft/sec.

Find the maximal value for the height (second coordinate) of the ball, the time for the maximum height, the coordinates of the position point of the ball for the maximum height, and the distance traveled by the ball from its initial position to the new position when it attains this maximal height.

Ans.

$$m\vec{a} =< 0, -mg > .$$

$$\vec{a} =< 0, -g > .$$

$$\vec{v} =< \frac{\sqrt{2}}{2}, -gt + \frac{\sqrt{2}}{2} >$$

$$\vec{p} =< \frac{\sqrt{2}}{2}t + 1, -gt^2/2 + \frac{\sqrt{2}}{2}t + 1 > .$$

Let h denote the maximum height. This occurs when $-gt + \frac{\sqrt{2}}{2} = 0$. Whence at time

$$t = \frac{\sqrt{2}}{2g}.$$

Thus

$$h = -\frac{g}{2}(\frac{\sqrt{2}}{2g})^2 + \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2g} + 1 = -\frac{g}{2}\frac{2}{4g^2} + \frac{2}{4g} + 1 = 1 + \frac{1}{4g}.$$

When the mass achieves this height its x-coordinate is

$$\frac{\sqrt{2}}{2}t + 1 = \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2g} + 1 = 1 + \frac{1}{2g}.$$

The distance d travelled is the length of the vector between $< 1, 1 >$ (the initial position) and $< 1 + \frac{1}{2g}, 1 + \frac{1}{4g} >$ (the position of maximum height). Thus

$$d = | < \frac{1}{2g}, \frac{1}{4g} > | = \frac{1}{2g}| < 1, 1/2 > | = \frac{\sqrt{5}}{4g}.$$