Problems for Math 210 - Review for Exam1, Spring 2016

1. (a) Given the vectors $\vec{v} =< 1, 2, 3 >$ and $\vec{w} =< 1, 1, 1 >$, find the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$ in three-space.

(b) For the vectors in part (a), find the length of the projection of $\vec{v}$ on $\vec{w}$ and the length of the projection of $\vec{w}$ on $\vec{v}$.

(c) For the vectors in part (a), give the equation of the plane through the origin that contains the coordinate points $(1, 2, 3)$ and $(1, 1, 1)$. Your answer should be of the form $ax + by + cz = 0$. Determine the values for a, b and c.

2. A ball of mass m is initially at the point $(1, 1)$ in the plane and is subject to a gravitational force of $< 0, -mg >$. The ball is given an initial velocity $< cos(\pi/4), sin(\pi/4) >$. Here $g = 32ft/sec^2$, and the velocity components are in ft/sec.

Find the maximal value for the height (second coordinate) of the ball, the time for the maximum height, the coordinates of the position point of the ball for the maximum height, and the distance traveled by the ball from its initial position to the new position when it attains this maximal height.

3. Find the equation of the line which is parallel to the line

$$\vec{r}(t) =< 1 + 3t, 2 - 6t, 3 + 4t >$$

and which passes through the point $P(1, 3, 7)$.

4. A particle is moving in space with acceleration described by the function $\vec{a}(t) =< 1, sin(3t), sin(7t) >$ At time t = 0 the particle is stationary and it is located at the origin (0, 0, 0).
(a) Compute the position function $\vec{r}(t)$.
(b) Compute the position vector of the particle at time $t = \pi/2$ seconds.
(c) Write down an integral that will give the total distance traveled by the particle during the first second of its travel. Do not evaluate.

5. Give an equation of the plane that contains the points $P(3, 3, 7), Q(2, 2, 2)$ and $R(0, 1, 0)$.

6. Are the planes given by the equations

$$3x + 4y + 5z = 1$$

and

$$3x + 4y - 5z = 137$$

orthogonal? Find the equations for the line of intersection of these two planes.

7. Consider the plane with equation

$$2x + 3y + z = 8.$$

Find vectors $\vec{P}, \vec{Q}, \vec{R}$ so that the set of points in this plane can be described by the set of all vectors of the form

$$\vec{P} + s\vec{Q} + t\vec{R}$$

where s and t are any real numbers.

8. A surface is given by the equation

$$x^2/4 + y^2/9 - 3z = 0.$$

(a) Find the equations of the $xy-$, $yz-$ and xz-traces of the surface.
(b) Sketch the surface.

9. Given a function of two variables

$$f(x, y) = \sqrt{x^2 - y^2 - 20},$$

find and sketch its domain. Find its level curve for $f(x, y) = 5$.