Math 215 -- Assignment 5

Part 1.

Background for this problem is in Chapter 14 of Eccles, but it is best for you to try the problem and then look at Chapter 14. Consider the following discussion.

Suppose that F:X -- P(X) is a well-defined function from a set X (any set) to its power set P(X).

Recall that P(X) is the set of subsets of X.
Thus for each x in X, we have F(x) is a subset of X.
This means that x might or might not be a member of F(x).

Let C(F) = {x in X| x is not a member of F(x)}.
Prove that C(F) is not of the form F(z) for any z in X.

This problem is rather abstract, and so you might like to do the following problem first. Before proving that C(F) is not of the form F(z) in general, take the following special case:
X = {1,2,3} and
F(1) = {2,3},
F(2) = {1,3},
F(3) = {1,2,3}.
Then what is C(F)? Do you see that C(F) is not of the form F(z) for any z in X? Fine.

Now make up some more examples of your own, of maps from a set to its power set, and construct the corresponding set C(F). You can use either finite or infinite sets in your examples. Here are a couple of examples to work with:
(1.) X = N = {1,2,...},
F(n) = {p | p is a prime number that divides n}
(note that a number p is prime if it is a natural number not equal to 1, such that the only divisors of p are 1 and itself.)
(2.) X = N,
F(1) = { },
F(2)= {1},
F(3) = {2},
F(4) = {1,2},
F{5} = {3},
F{6} = {1,3},
F{7} = {1,2,3},

...
The intent here is the F should be defined so that F makes a list of all the **finite** subsets of N. Can you devise an inductive definition of F? Find C(F).

Once you have proved this property of C(F), it is worth reflecting on what you have found. We will discuss this in class and you can read Chapter 14 in Eccles.

Part 2.

Write a new version of your proof that F(n+1) = F(n) + n +1 for F(n) equal to the number of regions into which the plane is divided by n lines configured so that each line intersects every other line once. Generalize your result to the function R(n) where R(n) equals the number of regions in three dimensional space that are cut out by a collection of n planes such that each plane intersects all of the other planes in one line for each plane (and the pattern of line intersections in each plane is a set of lines each intersecting all of the other lines). Show that R(n+1) - R(n) = F(n). Find a formula for R(n) by using discrete calculus. See the notes on discrete calculus on our site.

Part 3.

page 57: Try Problem #26. You are not yet required to write up this problem.
page87: 7.7, 7.8
page 99: 8.1,8.5
page 113: 9.1,9.4,9.5
page 116: 6.