Exam3 - Math 313 **- Fall** 2014

1. Prove that $1 = .9999\cdots$ by using only the definition of limit. Note that you will have to explain what is the meaning of an expression like $.999\cdots$ as a limit of a certain sequence.

2. (a) Given a sequence $\{a_n\}$ of real numbers, give the definition of the statement: "$lim_{n\longrightarrow\infty}\ a_n$ exists."

   (b)**True or False**: $lim_{n\longrightarrow\infty}\ \frac{n^3}{n^2+n^4} = 1$.

   (c)**True or False**: $lim_{n\longrightarrow\infty}\ \frac{(-1)^n}{2^n} = 0$.

3. (a) Give the definition of convergence for a series $S = \sum_{n=1}^{\infty} b_n$ where S is a real number and b_n is a real number for each natural number n.

   (b) Give an example of a series $\sum_{n=1}^{\infty} b_n$ that converges, but $\sum_{n=1}^{\infty} |b_n|$ diverges.

   (c)**True or False**: $\sum_{n=0}^{\infty}(-1)^n = 0$.

   (d)**True or False**: $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ converges and its value is $2/3$.

4. (a) **True or False**: The union of a countable collection of closed intervals in the real line is always a closed set.

   (b) Give a specific example of an *empty* intersection of an infinite collection of nested open intervals where each individual interval is non-empty.

   (c) Explain the definition of the Cantor Set and give at least one interesting fact about the Cantor set.

5. (a) Suppose that A is an open subset of the real line.

   True or False: There exists a point $x \in A$ and an $\epsilon > 0$ so that the open interval $(x - \epsilon, x + \epsilon)$ intersects A in only the point x.

   (b) **True or False**: There exist subsets of the real numbers that are both open and closed.

   (c) A function is said be bounded if the set of its function values is a bounded subset of the real line.

   True or False: A continuous real-valued function, defined on a closed interval is bounded on that interval.