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Mathematical bios

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Abstract *In this paper we report on a mathematical pattern that we call bios, and its generation by recursions of bipolar feedback. Bios is a newly found form of organization, that resembles chaos in its aperiodic pattern and its extreme sensitivity to initial conditions, but has additional properties (diversification, novelty, nonrandom complexity, life-limited patterning, 1/f power spectrum) found in natural creative processes, and absent in chaos. The process equation* $A_{t+1}=A_t+g_t \sin(A_t)$ *generates convergence to* π*, a cascade of bifurcations, chaos, bios and infinitation, as the value of the feedback gain* g_t *increases. In the complex plane, series generated by orthogonal process equations display fractal organic patterns.*

Defining bios

Bios is a pattern that we have discovered through examining both mathematical recursions (Kauffman and Sabelli, 1998) and biological, meteorological and economic data (Carlson-Sabelli *et al.*, 1995; Sabelli, 2000; Sabelli and Kauffman, 1999; Sabelli *et al.*, 1997). As natural creative processes, bios is nonstationary aperiodic series that displays:

(1) diversification [increased variance with the duration of the sample (Sabelli and Abouzeid, in press)];
(2) novelty meaning less recurrent than its randomized copy (Figure 1) (Sabelli, 2001a, b);
(3) nonrandom complexity (Sabelli, 2002);
(4) episodic patterns with a beginning and end ("complexes"), in contrast to random, periodic or chaotic series that show uniform configuration over time; and
(5) global sensitivity to initial conditions; chaos is only locally sensitive.

The statistical distributions of biotic series are multimodal and asymmetric, in contrast to symmetric random, periodic and chaotic series. We regard

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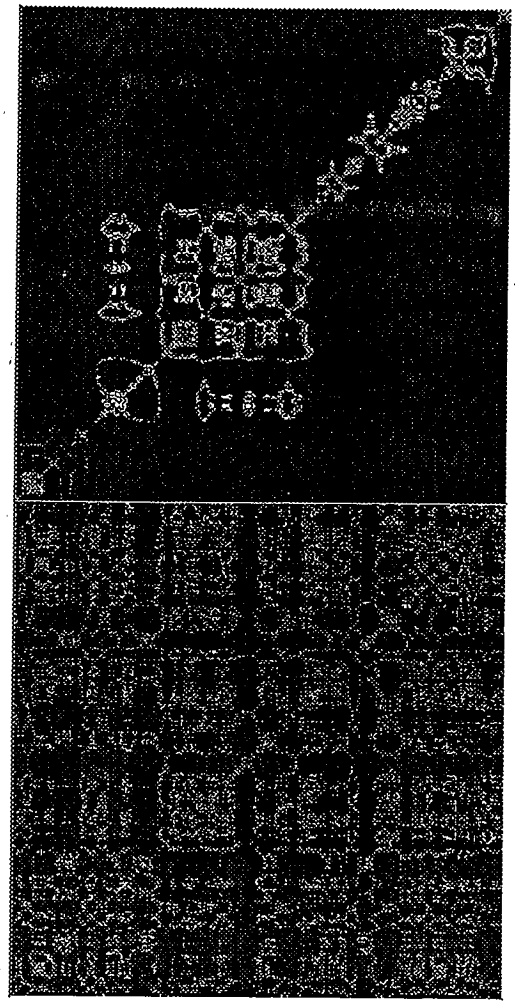

Figure 1.
Recurrence plots of biotic series (top) and of its shuffled copy (bottom). Note the time-limited patterns (complexes) in bios, and the increase in recurrences with shuffling (novelty)

asymmetry as an essential component of creativity, because cosmological evolution can be understood as a sequence of symmetry-breakings. Other noteworthy properties of bios are the $1/f$ power spectrum and the generation of Mandala patterns in complement plots (Figure 2). As chaos, bios is aperiodic, deterministic, and extremely sensitive to initial conditions. Chaos is bounded (Kaplan and Glass, 1995); bios may be bounded or not.

Diversification, novelty, nonrandom complexity, and episodic patterning are measurable properties found in bios and in creative natural processes, but not observed in random, periodic, or chaotic series. In contrast to simpler chaos, bios is creative. We thus regard bios as an authentically new phenomenon rather than as a subtype of chaos. The exemplar of bios is heart rate variation, just as the exemplar of chaos is turbulence.

Process equations

Bios is generated by the recursion

$$A_{t+1} = A_t + g \sin(A_t)$$

where A_t are real numbers. We call this equation the *process equation* (Kauffman and Sabelli, 1998). It models interactions that produce both positive (augmenting, synergistic) and negative (decreasing, antagonistic) change. Change is a function of the previous action, i.e. a feedback. The feedback is bipolar and diverse, spanning the range from plus to minus g through the continuity of the trigonometric function; g is the feedback gain. The circle map, widely discussed in the literature, computes the same recursion modulo π (Kaplan and Glass, 1995). The circle maps does not produce bios. Interesting variants are the process equation with delay

$$A_{t+1} = A_t + g \sin(A_{t-1})$$

that generates biotic series very close to those found in heartbeat data (Sabelli, 1999), and the kinetic process equation (Sabelli and Kauffman, 1999)

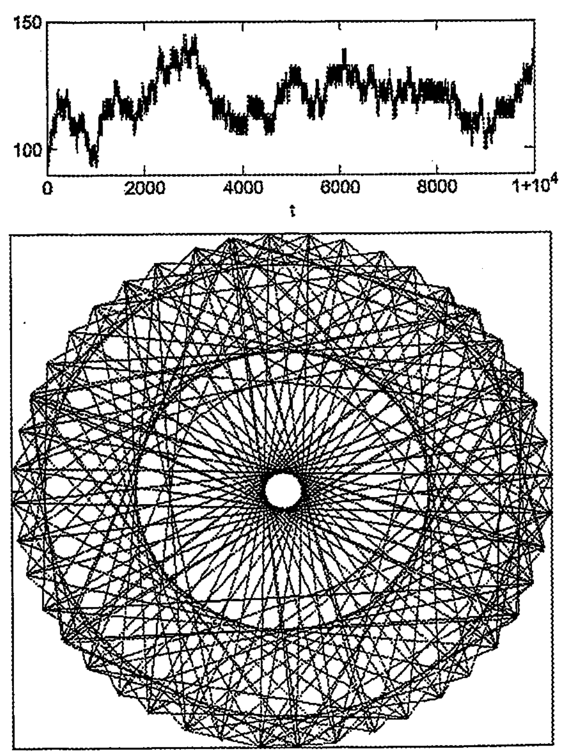

Figure 2.
Complement plot. Top: Biotic series generated by process equation with delay $A_{t+1} = A_t + g\sin(A_{t-1})$, $g = 3$, rounded to the nearest integer. Bottom: Complement plot generated by plotting the sine (y axis) and the cosine (x axis) of each term in a Cartesian plane and connecting successive term. The circular form imposed on the data reveals a surprising regularity that reminds us of Mandala archetypes. This pattern also appears in series of heartbeat intervals (Sabelli, 2000) but not in chaos

$$A_{t+1} = A_t + g_t \sin(A_t)$$

in which the gain is a function of time. We usually take $g_t = kt$, where k is a small constant and t is the number of iterations. As the gain g_t increases, the time series generates equilibrium, a cascade of bifurcations forming 2^n periods, chaos, and bios, a sequence of increasing complexity and amplitude (Figure 3).

For $g < 2$, the equation converges to an odd multiple of π (e.g. π for initial values between 0 and 2π). As the gain is increased the fixed point becomes unstable. At $g \geq 2$, a bifurcation generates asymmetric opposites that diverge as the gain increases. Of course, it is the computational error in the computer that allows the bifurcation to begin. If the computer were mathematically perfect, then the process would remain at the fixed point! Following the initial bifurcation, there is shift in each branch of the series that looks as a bifurcation in which only one outcome is visible (unifurcation). The polarity of the unifurcation can be reversed by changing k, the rate of increase in gain.

As g increases a cascade of period-doubling bifurcations generates 2^N periods, followed by a transition to chaos, analogous to that in the logistic equation $A_{t+1} = gA_t(1 - A_t)$. Initially, the chaotic regime overlaps with period 2, at variance with the description of mutually exclusive basins of attraction. "Period 2 chaos" can also be readily demonstrated in the logistic equation, although we are not cognizant of previous reports. Remarkably, we can experimentally verify that the bifurcation sequence of the logistic equation occurs inside the time series of the process equation (Figure 4). In the process recursion, however, we have the abrupt expansions never seen in logistic chaos. Process chaos is interspersed with periodicities, among which

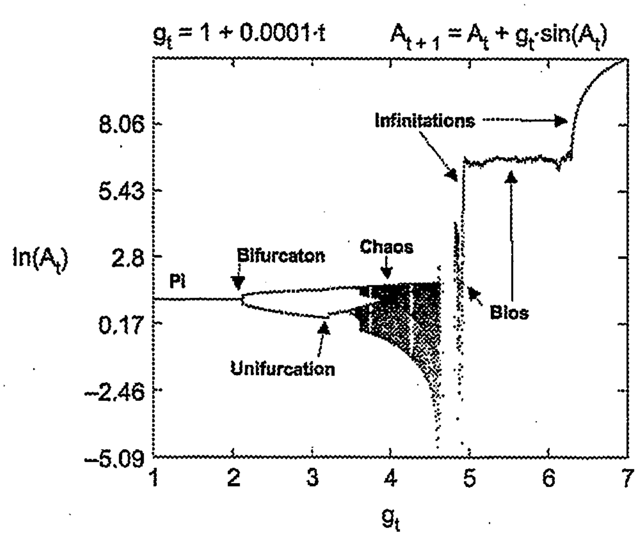

Figure 3.
Time series generated by the kinetic process equation. Note logarithmic scale

period 6 and period 4 are prominent. In contrast, period 3 is prominent in logistic chaos.

The range of A_t increases throughout the chaotic phase, but it remains within the basin of attraction, and is always smaller than the range of differences between successive terms. When the range reaches 2π ($g \leq 4.604$), the time series expands both positively and negatively, generating aperiodic biotic patterns resembling those observed with cardiac data. The range of A_t becomes much larger than ΔA_t. This disproportion between long duration change and moment-to-moment differences distinguishes bios from chaos. The transition from chaos to bios is best visualized in cobweb plots (Figure 5).

Aperiodic bios is periodically interrupted by bioperiod 2 when the gain equals odd multiples of π (half rotation). From this new pair of opposites, a new biotic phase emerges. Chaos does not show periodic repetitions of period 2. Bioperiods differ from other periods in being sensitive to initial conditions. Aperiodic bios also is periodically interrupted by flights toward positive or negative infinity (infinitation) when the gain equals an even multiple of π (full rotation) and at few other critical values. As g increases further, new biotic series emerge, and further infinitations follow – a mathematical metaphor for death and renewal, essential features of living processes.

This sequence from one initial state, successive bifurcations, and the generation of complex chaotic and biotic patterns resembles cosmological evolution and embryological development. This series illustrates the greater creativeness of bipolar interactions, both synergic and antagonistic, in comparison to simple positive or negative feedback. Process development is very flexible, allowing any initial value, and any value or sign for the g

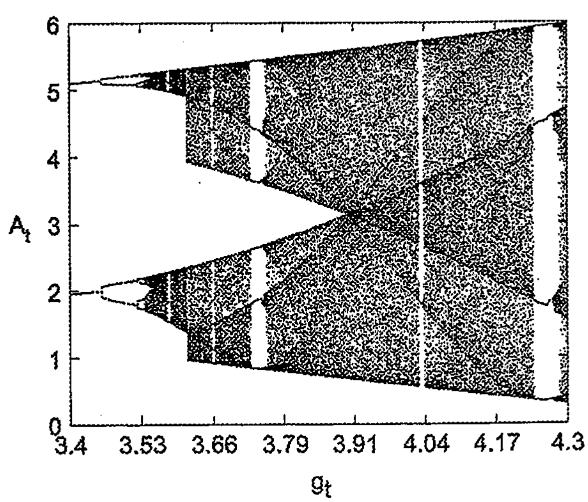

Figure 4.
Detail of process chaos, showing period 2 chaos in which each branch shows a logistic pattern that ends with the expansion of chaos that appears to continue the virtual branch of the unifurcation. There are periods 6 before and after this expansion. After the unification of chaos (end of period 2), there is a narrow period 3, and a prominent period 4

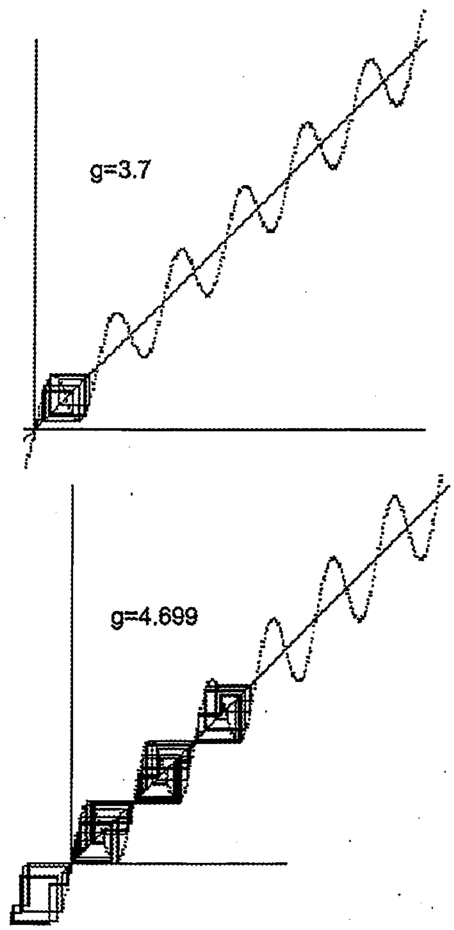

Figure 5.
Cobweb plots during chaos (top) and bios (bottom). Each trajectory is generated with a constant g. The sinusoidal envelope in the cobweb plot portrays the generator of the aperiodic biotic process. The plot has a dotted wavy figure that is the graph of $y = f(x) = x + g\sin(x)$ where x denotes the horizontal axis and y denotes the vertical axis. We draw lines from (x, x) to $(x, f(x))$ and from $(x, f(x))$ to $(f(x), f(x))$. The iterates of the function $f(x)$ appear as the off-diagonal points in the cobweb

parameter, and generates biotic patterns beyond chaos. Logistic development requires initial values between 0 and 1, and flights into infinity after chaos.

Sensitivity and reversibility

Bios, bioperiods, and infinitations show global sensitivity to initial conditions, meaning that the entire series is displaced by a small change in initial value (Figure 6 top). In chaos, initial conditions change the trajectory but not the overall distribution of the data (Figure 6 bottom). Bios, bioperiods, and infinitations also show extreme sensitivity to changes in the rate k of change of the gain.

If the gain first increases and then decreases, the time series evolves from an initial steady state to greater complexity, and then devolves back to simpler patterns (Figure 7). If the series evolves up to chaos, a subsequent decrease in the gain leads back to the initial steady state. In

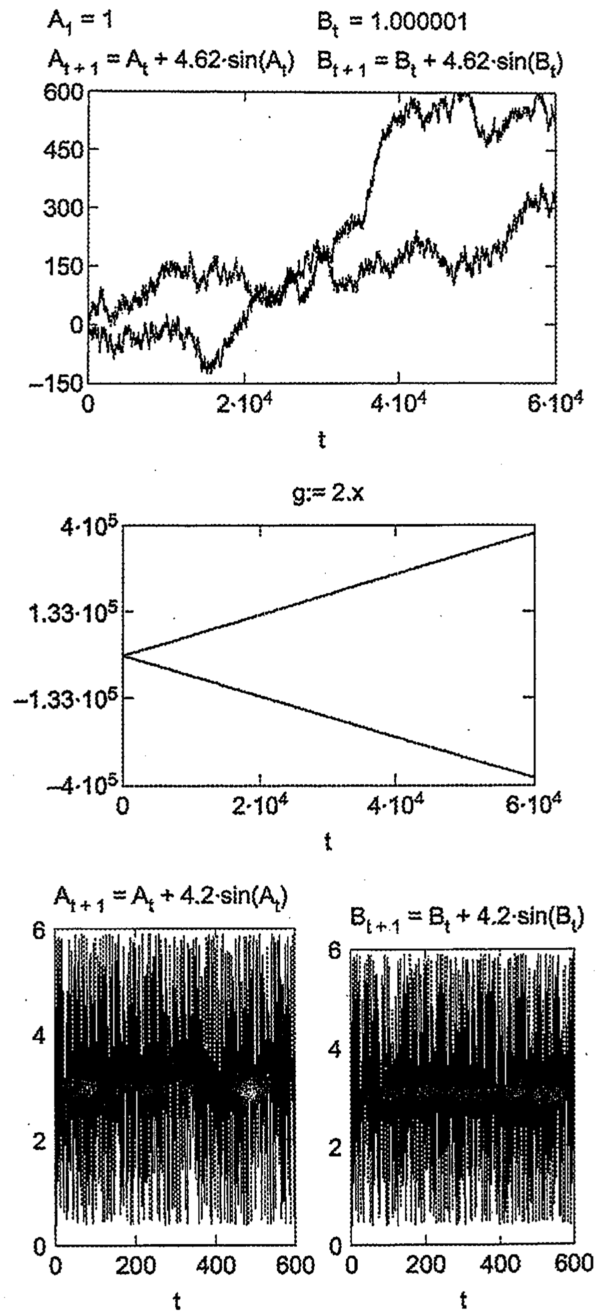

Figure 6.
Global sensitivity to initial conditions in bios (top) and infinitation (middle). Local sensitivity in chaos (bottom)

contrast, if the series reaches the biotic regime, a subsequent decrease in gain does not lead back to the starting point. One is tempted to relate this mathematical irreversibility of bios, absent in chaos, to the irreversibility of physical processes.

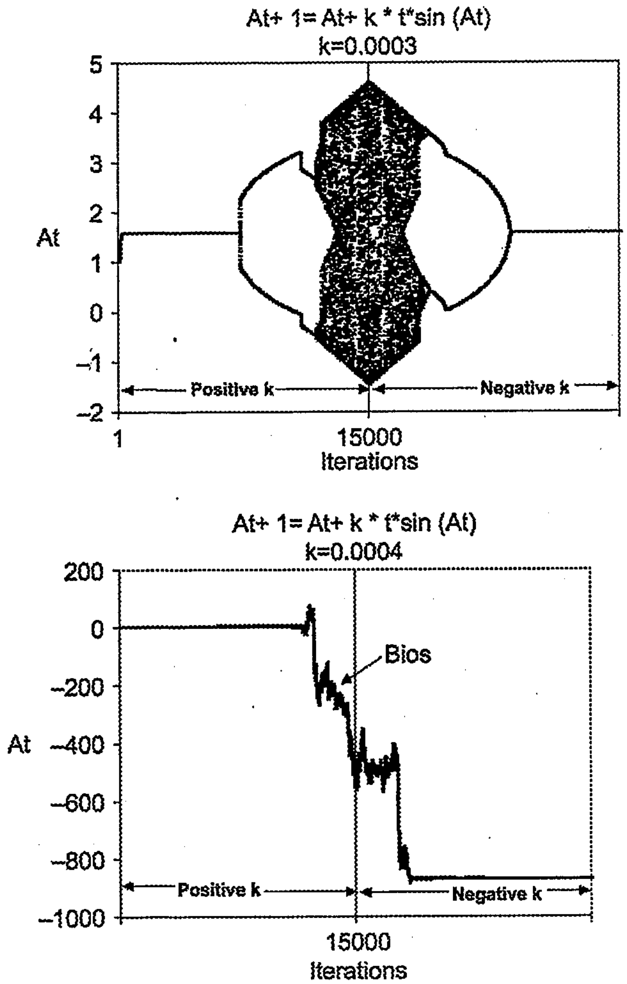

Figure 7.
Time series computed with a positive increment in gain for 15,000 iterations and then a negative value for *k*

Differencing bios, integrating chaos

The time series of differences between successive members is chaotic. *Biotic patterns in natural processes will be mistakenly identified as chaotic when data are differenced prior to analysis*. Integrating chaotic series (logistic chaos, process chaos, shift map, sine map, Rossler, Ikeda, and Henon attractors) does not generate bios. One may generate bios-like series by integrating the Lorenz attractor, the chaotic series generated by $A_{t+1} = g\sin(A_t)$ or $A_{t+1} = g\cos(A_t)$, or other chaotic series after rescaling them to mean 0. Integrated chaos shows diversification and novelty, but do not show time-limited complexes characteristic of mathematical bios and of biological or economic data.

Co-creating equations

The process equation models Heraclitus' notion that the interaction of opposites generates evolution (Sabelli and Kauffman, 1999). To study the phenomenon of co-creation (Sabelli, 2001a, b), we are exploring systems of two, three and many interacting process equations. We find that circles of equations enter into stable chaos, while cascades of equations generate progressively more complex patterns.

The recursion shown in Figure 8

$$A_{t+1} = A_t + B_t \cos(A_t)$$

$$B_{t+1} = B_t + A_t \sin(B_t)$$

is a self-contained two dimensional process consisting in two interlocked process equations where the gain of each equation is the output of the other equation. This mutuality gives rise to a very beautiful attractor (Sabelli, 1999).

In the co-creating equations

$$A_{t+1} = A_t + g\,B_t \sin(B_t)$$

$$B_{t+1} = B_t + h\,A_t \cos(A_t)$$

the gain and feedback of each recursion comes from the other one. For specific values as shown in Figure 9, we get a most extraordinary development of an organic-like pattern that displays fractal repetition at successive levels of magnification.

In summary, biotic and organic patterns can be generated by simple equations that model bipolar and diverse opposition, suggesting how synergic and conflictual interactions in nature may contribute to creative evolution.

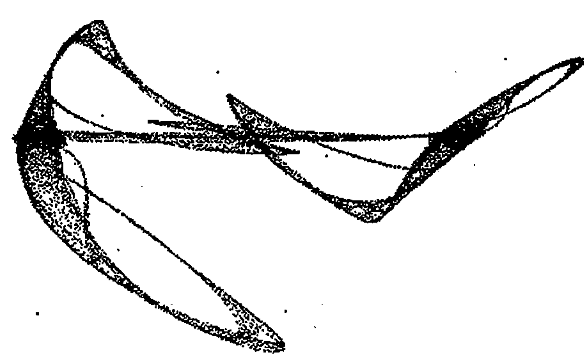

Figure 8.
The Sabelli attractor (see text)

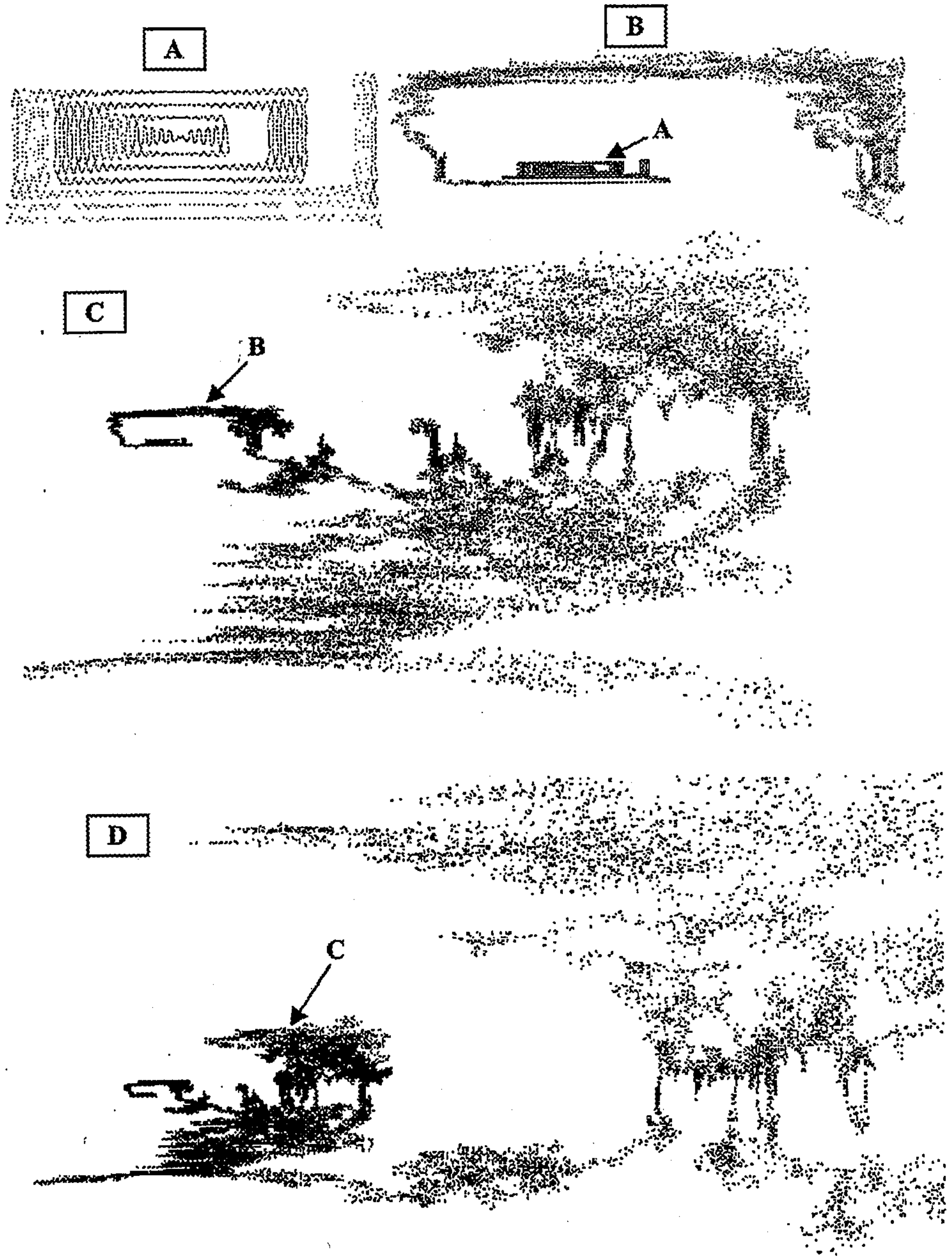

Figure 9.
The generation of organic forms by the interaction of two process equations. X axis: $A_{t+1} = A_t + 0.1\,B_t \sin(B_t)$. Y axis: $B_{t+1} = B_t + 0.01\,A_t \cos(A_t)$. Initial values $A_1 = 6.3734761$; $B_1 = 0.001$. The progressive development of complex pattern with iteration is presented in the sequence of XY graphs A, B, C, and D. Note how each larger pattern contains and repeats at a larger scale the previous ones (fractality)

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