The Peano Axioms for the Natural Numbers
by LK

The natural numbers consist in a set N that satisfies the following axioms:

A1. There is a member of N called 1.
A2. If x is a member of N, then there exists a unique element x' of N, called the *successor of x.*
A3. For all x in N, x' is not equal to 1.
A4. Given elements x and y in N, if x' = y' then x = y.
A5. Suppose we are given a subset M of N with

I. 1 belongs to M, and
II. If x is in M, then x' is in M.

We call such a subset M of N an *inductive subset of N.*
This axiom asserts that following statement.
An inductive subset of M of N is equal to all of N.
(This is the Peano version of the Principle of Mathematical Induction.)

Here is a sequence of problems that begin the construction of the usual propoerties of the natural numbers, based on the Peano axioms. If not stated directly, we assume that x,y,.. are members of N.

1. x not equal to y implies x' not equal to y'.

2. x' is not equal to x.

3. x not equal to 1 implies that there exists a unique u such that x = u'.

4. To every pair x, y in N, we may assign in exactly one way, a natural number r, called x + y, such that
(a) x + 1 = x' for all x in N.
(b) x + y' = (x+y)' for all x and y in N.

5. (x + y) + z = x + (y + z) for all x,y, z in N.

6. x + y = y + x for all x and y in N.

7. y is not equal to x + y for any x and y in N.

8. y not equal to z implies that x + y is not equal to x + z for any x,y, z in N.

9. Given x and y in N, then either x = y or x = y + u or y = x + v for some u and v in N.