

Worksheet 12: Practice Final Exam

Name:

December 7, 2014

1. Evaluate the following integrals:

(a) $\int_1^{\ln 10} \frac{e^{\ln x}}{x} dx$

(b) $\int [4\sin(\cos(y))\sin y + y^6] dy$

2. You have 40 feet of fencing and you want to create a rectangular pig pen against the side of your house. This means you are using your fencing for three of the four edges. What are the dimensions of the pen with the greatest area that you could build?

3. Let $f(x) = x^3 + 3x + 4$. Use the definition of the derivative to find $f'(1)$.

4. Let $F(x) = \int_0^{x^2} e^{2t+1} dt$. What is $F'(x)$?

5. A thin sheet of ice is in the form of a perfect circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of $0.5\text{m}^2/\text{sec}$, at what rate is the radius decreasing when the area of the circle is 12m^2 ?

6. Use linear approximations to estimate the value of $e^{0.1}$.

7. Consider the continuous function $f(x)$ where $f(x) > 0$ for $x \geq 0$, and $f(x) < 0$ for $x < -2$, and $\int_{-2}^0 f(x) = 0$.

(a) Graph the function.

(b) Must the function go through the line $y = 0$? Why or why not?

(c) Must the function go through the line $x = 0$? Why or why not?

8. Evaluate the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{e^x}{x^5}$

(b) $\lim_{t \rightarrow 0} \frac{\sin(t)}{t}$

9. Find and classify the local extrema of the following functions:

(a) $f(x) = \sin(x)\cos(x)$

(b) $g(x) = \frac{\sin(x)}{e^{\ln x}}$

10. Explain how Riemann Sums are used to approximate the area under a curve. Be sure to talk about the relationship between area, integrals, and Riemann Sums. In your explanation, estimate the area under the curve $f(x) = x^2$ from 0 to 4 using a left and right Riemann Sum.