

1. Construct a single function $f(x)$ with all of the following properties:

(i) $f(x) \geq 0$ for $x \geq 0$.

(ii) $g(x) = \int_0^x f(t) dt$ is concave up for $x \geq 0$

(iii) $f(x)$ is concave down.

(Start by drawing such a function. Then try to write a formula)

2. Compute the definite integrals:

$$(a) \int_{-2}^2 x^4 - 6 dx \quad (b) \int_1^{e^3} \frac{1}{x} dx \quad (c) \int_{-\pi}^{2\pi} |\sin(x)| dx$$

3. Use substitution to calculate the following integrals:

$$(a) \int_1^2 \frac{x^3}{(1+x^4)^{1/3}} dx \quad (b) \int_0^{\sqrt{\pi}} x \cos(x^2) dx \quad (c) \int \cos^3(x) \sin(x) dx.$$

4. Let

$$R(t) = \int_1^{t^2} \ln(\arctan(4x^3 - 6x)) \cot(x) dx.$$

Find $R'(t)$.