

u-Substitution

1. Evaluate the following integrals:

$$(a) \int_1^{\ln 10} \frac{e^{\ln x}}{x} dx$$

$$(b) \int \sin(\cos(x)) \sin(x) dx$$

$$(c) \int_1^3 x\sqrt{10-x^2} dx$$

$$(d) \int \frac{e^x}{1+e^{2x}} dx$$

Optimization

1. Suppose you want to construct a window in the shape of a rectangle topped with a semi-circle. Find the window with maximal area subject to the constraint that the window's perimeter must be 6 feet.

2. You have 40 feet of fencing to create a rectangular pig pen. You use the wall of your house for one side of the fence. What is the maximal area pig pen you can create (you may assume your house is very long).

Limits

1. Find all (vertical and horizontal) asymptotes of the following functions:

$$(a) f(t) = t^{-2}$$

$$(b) q(x) = \frac{x^2 - 4x + 3}{x^2 - 5x + 6}$$

$$(c) p(s) = \frac{1}{\ln(s)}$$

2. Evaluate the following limits:

$$(a) \lim_{x \rightarrow \infty} \frac{3x^9 - 12x^3 + 1}{7x^4 - 6x^9}$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin(x)}{e^x - 1}$$

$$(c) \lim_{x \rightarrow 0} \frac{|x|}{x}$$

Fundamental Theorem of Calculus

1. State the Fundamental Theorem of Calculus (both flavors). What properties must a function $f(x)$ have in order for the fundamental theorem of calculus to apply?

2. Use the fundamental theorem of calculus to evaluate the following expressions:

$$(a) \int_0^4 9x^2 - 6x + 1 dx$$

$$(b) \int_{-3}^3 x^3 + 4x dx$$

$$(c) \int_0^\pi \cos(x) dx$$

$$(d) \frac{d}{dt} \int_{17}^t e^{(x^2)} dx$$

$$(e) \frac{d}{dq} \int_{2q}^5 e^{8x} \tan(x) dx$$

$$(f) \frac{d}{dy} \int_{2y}^{\cos(3y^2)} e^{(-1/x^2)} dx$$

Related Rates

1. Gregor Clegane fills a cylindrical tank with some lava. Suppose the tank has radius 2 meters. If Gregor is pouring lava at a rate of 3 cubic meters per second, at what rate is the lava level rising? (You may assume the tank is made of mithril.)

Derivatives

1. Compute the derivative of the following functions:

$$(a) \cos(\tan(x)) \qquad (b) \sqrt{\frac{e^x - 4x}{7x^3 + 4x}} \qquad (c) \tan^{-1}(\ln(x))$$

2. Use implicit differentiation to find dy/dx , where $xy^2 + \cos(y) - \sin(x) = \ln(2y)$.

The Shape of Functions

1. Let $f(x) = -x^3 - 4x^2 + 3x + 18$.

(a) Find and classify all critical points of $f(x)$.

(b) On what intervals is $f(x)$ increasing? Decreasing?

(c) Where is $f(x)$ concave up? Concave down?

(d) Sketch a graph of $f(x)$.

2. Draw a function $g(x)$ satisfying ALL of the following: $g(-2) = 0$; $g'(x) > 0$ for all x ; $g''(-1) = 0$; $g''(x) > 0$ for $x < -1$; $g''(x) < 0$ for $x > -1$.

Integration

1. Suppose f and g are continuous functions and

$$\int_a^b f(x) dx = \int_a^b g(x) dx.$$

True or false: $f(x) = g(x)$.

2. Let

$$f(x) = \begin{cases} 1 & x > 3 \\ 2 & x \leq 3 \end{cases}$$

Compute $\int_0^5 f(x) dx$.

3. Compute the following integrals:

(a) $\int xy^2 dx$

(b) $\int xy^2 dy$