Limits

1. Let

$$f(x) = \begin{cases} 1 & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$

For which a does $\lim_{x\to a} exist?$

2. Compute each of the following limits:

(a)
$$\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8}$$
 (b) $\lim_{x \to 4} \frac{3 - \sqrt{x+5}}{x-4}$ (c) $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

- **3.** Let $g(t) = t^2 \ln(t^2)$.
- (a) What is the domain of g?
- (b) Sketch a graph of g.
- (c) Using your graph, demonstrate that $-|t| \le t^2 \ln(t^2) \le |t|$ for t in [-1, 1].
- (d) Evaluate $\lim_{t\to 0} g(t)$.

4.

- (a) Compute $\lim_{x \to 1} 9x^2 + 3x + 7$.
- (b) Compute $\lim_{t\to 3} t^2 + t + 7$. Say something intelligent about your answers to (a) and (b).
- (c) Compute $\lim_{x\to 0} \frac{\sin(x)}{x}$. You may reference your textbook if you like.
- (d) Carefully contemplate your answers to (a)-(c), then compute $\lim_{x\to 0} \frac{\sin(4x)}{3x}$.