

2. Compute each of the following limits:

$$(a) \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8} \quad (b) \lim_{x \rightarrow 4} \frac{3 - \sqrt{x+5}}{x-4} \quad (c) \lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

Solution: Each of these limits is a very common type of problem you'll encounter in calculus, and the solution for each generalizes pretty well. Notice that simply plugging in the limit doesn't tell you anything because you get an expression of the form $\frac{0}{0}$. What we have to do is simplify algebraically, then evaluate the limit. Notice that it helps a lot to be able to factor polynomials.

(a) You should notice that this problem involves some ugly fractions. Your first step should be to get the numerator into a nicer form.

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8} &= \lim_{x \rightarrow -2} \frac{\left(\frac{2+x}{2x}\right)}{(x+2)(x^2 - 2x + 4)} \\ &= \lim_{x \rightarrow -2} \frac{2+x}{2x(x+2)(x^2 - 2x + 4)} \\ &= \lim_{x \rightarrow -2} \frac{1}{2x(x^2 - 2x + 4)} \\ &= -\frac{1}{48}. \end{aligned}$$

(b) Whenever you see a square root in the numerator or the denominator, your instinct should be to multiply by the algebraic conjugate.

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{3 - \sqrt{x+5}}{x-4} &= \lim_{x \rightarrow 4} \frac{3 - \sqrt{x+5}}{x-4} \cdot \frac{3 + \sqrt{x+5}}{3 + \sqrt{x+5}} \\ &= \lim_{x \rightarrow 4} \frac{9 - (x+5)}{(x-4)(3 + \sqrt{x+5})} \\ &= \lim_{x \rightarrow 4} \frac{4-x}{(x-4)(3 + \sqrt{x+5})} \\ &= \lim_{x \rightarrow 4} \frac{-(x-4)}{(x-4)(3 + \sqrt{x+5})} \\ &= \lim_{x \rightarrow 4} \frac{-1}{3 + \sqrt{x+5}} \\ &= -\frac{1}{6}. \end{aligned}$$

- (c) In some ways this is the most straightforward: factor the numerator and denominator and simplify. You'll notice that all the terms that go to zero will disappear. What happens when they don't? What if the polynomial in the numerator is nonzero when $x = 2$ but the one in the denominator is zero? We'll take about this in the next couple of weeks.

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{(3x + 5)(x - 2)}{(x - 2)(x + 2)} \\ &= \lim_{x \rightarrow 2} \frac{3x + 5}{x + 2} \\ &= \frac{11}{4}.\end{aligned}$$