Calculus I | # Midterm 1 Practice | **Spring 2015**

1.(**) Let $f(x) = \sqrt{x+4}$. Compute the derivative of $f(x)$ using the definition of the derivative as the limit of a difference quotient.

2.(*) Compute the derivative:

(a) $x^5 - 2x^4 + 3x - 1$ (b) $\cos(x)\sin(x)$ (c) $\frac{x^2-3}{\sin(x)}$

(d) $(x^7 - 2x)(x^3 + 4x + 1)(5x^2 + x + 1)$ (e) $\cos(x)\sin^2(x)$ (f) $\frac{1}{\tan(x)}$

3.(**) Compute the limit:

(a) $\displaystyle\lim_{x\to\infty} \frac{2x^4 - 3x^2 + 1}{-3x^4 - x}$ (b) $\displaystyle\lim_{x\to 1} \sqrt{e^{x-1} - x} \cdot \sin(1-x)$ (c) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x^4 + 25}}{x^2 + 3x + 1}$

4.(**) Let $s(t) = t^2 - 4t + 1$ describe the position of an object after t seconds. At what time is the instantaneous velocity of the object equal to the average velocity of the object on the interval $[2, 5]$?

5.(**) Let

$$f(x) = \begin{cases} 3x^2 & x \geq 0 \\ ax + b & x < 0 \end{cases}$$

For what values of a, b is $f(x)$ continuous? For what values of a, b is $f(x)$ differentiable?

6.(**) Draw a graph of one single function $f(x)$ such that each of the following properties is satisfied: (i) $f(x)$ is continuous (ii) $f(2) = 3$ (ii) $f'(2) = 0$ (iii) $f'(x) < 0$ for $x < 2$ (iv) $\lim_{x\to-\infty} f(x) = 0$

7.(**) let $f(x) = \dfrac{x^3 + 2x^2 - x - 2}{x^2 - 4}$. Identify all vertical asymptotes and compute the one-sided limits for each.

8.(*) Let $f(t) = t^2 + \sin(t)$.

(a) What is the average rate of change of $f(t)$ between 0 and π?

(b) What is the instantaneous rate of change of $f(t)$ at $t = \pi/2$?

9.(**) Let $f(x) = \sin(x) + \dfrac{x^\pi}{\pi^2}$. What is the equation of the line tangent to $f(x)$ at $x = \pi$?

10.(***) Let $f(x) = \dfrac{|x+3|}{x-1}$. Where is f continuous? Where is f differentiable? Show that there is a point a such that f is continuous at a but not differentiable at a. Compute:

$$\lim_{h\to 0^-} \frac{f(a+h) - f(a)}{h} \qquad \lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h}$$