Review 2 Solutions

1. Find all values of x that are NOT in the domain of the following functions.

   $f(x) = \frac{3x}{2x-10}$ $\qquad$ $g(x) = \frac{x^2+2x-63}{x^2-49}$

   To answer these, we can not let the denominators be zero, so any x value that makes the denominator zero is NOT in the domain.

   $2x-10=0 \quad x=5$ is not in the domain. $\qquad$ $x^2-49=0 \quad x=\pm 7$ are not in the domain.

2. Simplify.

   $\frac{7(2w+5)(w+6)}{21(w+4)(2w+5)}$ $\qquad$ $\frac{x-8}{x^2-64}$ $\qquad$ $\frac{4u^2-100}{u^2-8u+15}$

   $\frac{w+6}{3(w+4)}$ $\qquad$ $= \frac{x-8}{(x+8)(x-8)} = \frac{1}{x+8}$ $\qquad$ $= \frac{4(u+5)(u-5)}{(u-5)(u-3)} = \frac{4(u+5)}{u-3}$

3. Perform the operation and simplify.

$\frac{2y}{3a}\cdot\frac{9ay}{10y^5}$ $\qquad$ $\frac{4x-20}{45x-40}\cdot\frac{9x-8}{2x-10}$ $\qquad$ $\frac{x-1}{x^2-x-6}\cdot\frac{4x+8}{x-2}$ $\qquad$ $\frac{x^2-3x+2}{x^2+5x+6}\div\frac{4x-8}{x+2}$

$\frac{3}{5y^3}$ $\qquad$ $= \frac{4(x-5)}{5(9x-8)}\cdot\frac{9x-8}{2(x-5)} = \frac{2}{5}$ $\qquad$ $= \frac{x-1}{(x-3)(x+2)}\cdot\frac{4(x+2)}{x-2} = \frac{4(x-1)}{(x-3)(x-2)}$ $\qquad$ $= \frac{(x-2)(x-1)}{(x+3)(x+2)}\cdot\frac{x+2}{4(x-2)} = \frac{x-1}{4(x+3)}$

$\frac{c^2-9c}{c^2-c-12}+\frac{3c+8}{c^2-c-12}$ $\qquad$ $\frac{5}{6y}-\frac{9}{8y^2}$ $\qquad$ $\frac{2}{x-5}-\frac{3}{x+4}$ $\qquad$ $\frac{4}{3x^2+2x-1}+\frac{2}{3x^2-4x+1}$

$= \frac{c^2-6c+8}{c^2-c-12} = \frac{(c-4)(c-2)}{(c-4)(c+3)} = \frac{c-2}{c+3}$ $\qquad$ $= \frac{20y}{24y^2}-\frac{27}{24y^2} = \frac{20y-27}{24y^2}$ $\qquad$ $= \frac{23-x}{(x-5)(x+4)}$ $\qquad$ $= \frac{4(x-1)+2(x+1)}{(3x-1)(x-1)(x+1)} = \frac{2}{(x+1)(x-1)}$

$\frac{\frac{15s^4}{3t^5u^3}}{\frac{5rs^2}{9t^2u}}$ $\qquad$ $\frac{7-\frac{2}{5y}}{3-\frac{2}{5y}}\cdot\frac{5y}{5y}$ $\qquad$ $\frac{1-\frac{3}{x+6}}{\frac{9}{x+6}+x}\cdot\frac{(x+6)}{(x+6)}$ $\qquad$ $\frac{1}{u^{-1}-v^2}$

$\frac{15s^4}{3t^5u^3}\cdot\frac{9t^2u}{5rs^2} = \frac{9s^2}{rt^3u^2}$ $\qquad$ $= \frac{35y-2}{15y-2}$ $\qquad$ $= \frac{x+6-3}{9+x^2+6x} = \frac{x+3}{(x+3)(x+3)} = \frac{1}{x+3}$ $\qquad$ $= \frac{1}{\frac{1}{u}-v^2}\cdot\frac{u}{u} = \frac{u}{1-uv^2}$

4. Evaluate

   $256^{\frac{1}{4}}$ $\qquad$ $27^{\frac{1}{3}}$ $\qquad$ $(-8)^{\frac{1}{3}}$ $\qquad$ $81^{-\frac{1}{4}}$

   $= \sqrt[4]{256} = 4$ $\qquad$ $= \sqrt[3]{27} = 3$ $\qquad$ $= \sqrt[3]{-8} = -2$ $\qquad$ $= \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$

5. Simplify.

   $w^{\frac{5}{7}}\cdot w^{\frac{3}{4}}$ $\qquad$ $\frac{u^{\frac{1}{2}}}{u^{\frac{6}{7}}}$ $\qquad$ $\sqrt{27u^{14}}$ $\qquad$ $\sqrt{54x^{13}}$ $\qquad$ $\sqrt{8t^5y^8}$ $\qquad$ $\sqrt[3]{40t^8w^3}$

   $w^{\frac{20}{28}+\frac{21}{28}} = w^{1\frac{13}{28}} = w\sqrt[28]{w^{13}}$ $\qquad$ $u^{\frac{7}{14}-\frac{12}{14}} = u^{-\frac{5}{14}} = \frac{1}{\sqrt[14]{u^5}}$ $\qquad$ $3u^7\sqrt{3}$ $\qquad$ $3x^6\sqrt{6x}$ $\qquad$ $2t^2y^4\sqrt{2t}$ $\qquad$ $2t^2w\sqrt[3]{5t^2}$

   $\sqrt{75}-3\sqrt{27}$ $\qquad$ $4z\sqrt{32z}+\sqrt{18z^3}$ $\qquad$ $\sqrt{5z}\cdot\sqrt{7z}$ $\qquad$ $\sqrt[3]{3\cdot 4u^2}\cdot\sqrt[3]{3^2u^5}$

   $= 5\sqrt{3}-9\sqrt{3} = -4\sqrt{3}$ $\qquad$ $= 16z\sqrt{2z}+3z\sqrt{2z} = 19z\sqrt{2z}$ $\qquad$ $= z\sqrt{35}$ $\qquad$ $= \sqrt[3]{3^3\cdot 4u^7} = 3u^2\sqrt[3]{4u}$

$(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})$ $\qquad$ $(x+2\sqrt{2})^2$ $\qquad$ $\sqrt[4]{y}\cdot\sqrt[3]{y^2}$

$x+\sqrt{2x}-\sqrt{2x}+2=x-2$ $\qquad$ $x^2+2x\sqrt{2}+2x\sqrt{2}+8=x^2+4x\sqrt{2}+8$ $\qquad$ $y^{\frac{1}{4}+\frac{2}{3}}=y^{\frac{3}{12}+\frac{8}{12}}=y^{\frac{11}{12}}=\sqrt[12]{y^{11}}$

6. Solve. Remember to check your solutions.

   $\sqrt{3y+18}+2=5$ $\qquad$ $\sqrt{5x+10}=\sqrt{7x-12}$ $\qquad$ $\sqrt{11y-30}=y$ $\qquad$ $u-5=\sqrt{49-8u}$

   Isolate the radical, and then square both sides of the equation. Check your solutions. Most of these work, but the last problem has two possible solutions $u=6,-4$, but when you check them, only $u=6$ is the correct solution.

   $y=-3$ $\qquad$ $11=x$ $\qquad$ $y=5,6$ $\qquad$ $u=6$

7. Perform the operation and simplify. Write your solution in $a+bi$ form.

   $(6-2i)+(4+3i)$ $\qquad$ $(3-7i)-(5+4i)$ $\qquad$ $(-3+6i)(-4+3i)$ $\qquad$ $\dfrac{4-2i}{2-5i}\cdot\dfrac{2+5i}{2+5i}$

   For the last two problems, you will use the fact that $i^2=-1$ to simplify them.

   $10+i$ $\qquad$ $-2-11i$ $\qquad$ $12-9i-24i+18i^2=-6-33i$ $\qquad$ $\frac{8+20i-4i-10i^2}{4-25i^2}=\frac{18}{29}+\frac{16}{29}i$

8. Solve. You may need to use the quadratic formula.

   $(v-7)^2-32=0$ $\qquad$ $(w+9)^2-45=0$ $\qquad$ $2x^2+5x-1$

   $v=7\pm4\sqrt{2}$ $\qquad$ $w=-9\pm3\sqrt{5}$ $\qquad$ $x=\frac{-5\pm\sqrt{33}}{4}$

   The third problem had a typo. It should've been set equal to zero.

   $4x^2-9x+3=0$ $\qquad$ $3x^2+5x=3$ $\qquad$ $2x^2-3x+6=0$

   $x=\frac{9\pm\sqrt{33}}{8}$ $\qquad$ $x=\frac{-5\pm\sqrt{61}}{6}$ $\qquad$ $x=\frac{3\pm i\sqrt{39}}{4}$

9. A model rocket is launched with an initial upward velocity of 235 $\frac{ft}{sec}$. The rocket's height h (in feet) after t seconds is given by $h=235-16t^2$.

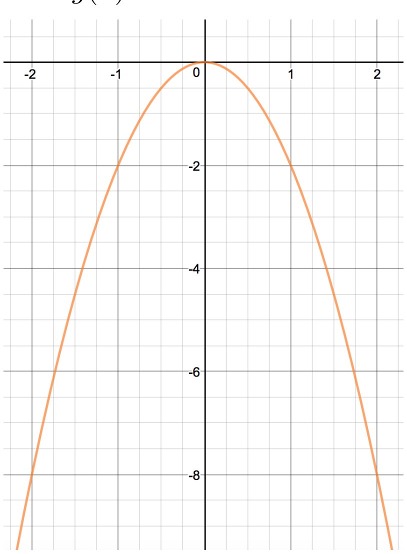
   Find all values of t for which the rocket's height is 151 ft.

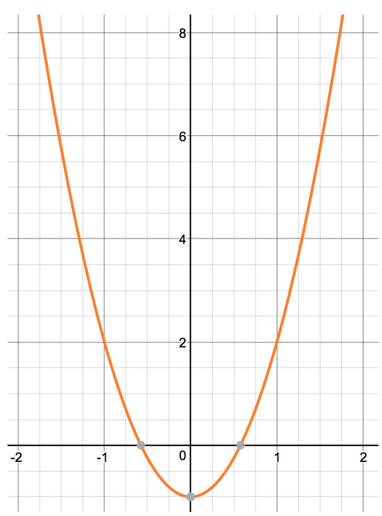
   This problem also had a typo. The equation should've been $h=235t-16t^2$. However, since the original does not say that, I will give the solution according to the original.

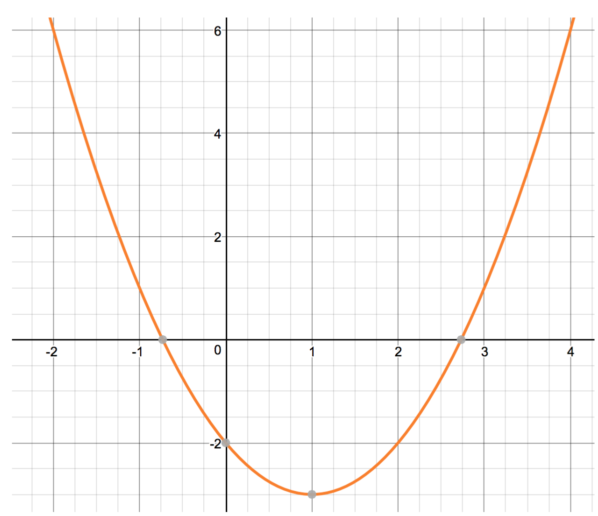
   $151=235-16t^2$ $\qquad$ $-84=-16t^2$ $\qquad$ $\frac{21}{4}=t^2$ $\qquad$ $\frac{\sqrt{21}}{2}=t$

10. Graph the following functions. Make sure to label the vertex.

   $g(x)=-2x^2$ $\qquad$ $h(x)=3x^2-1$ $\qquad$ $y=(x-1)^2-3$





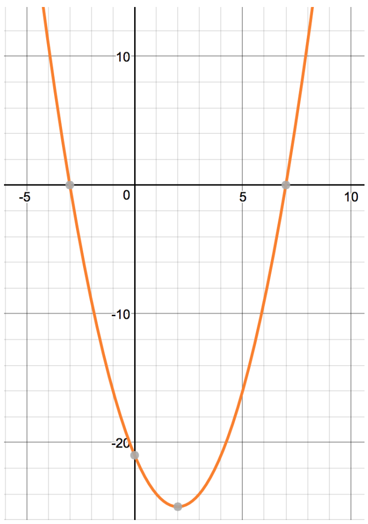


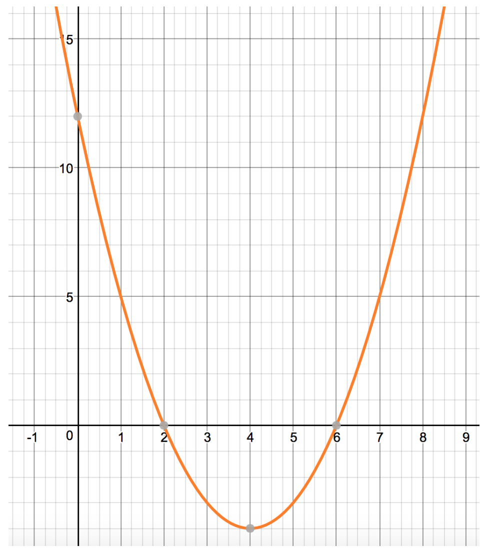
11. State the vertex and x-intercepts of the following functions, then use them to graph the function.

$y = x^2 - 4x - 21$ $\qquad\qquad$ $y = x^2 - 8x + 12$

Use the fact that the x-value of the vertex is $\frac{-b}{2a}$, and then plug this back in to get the y-value of the vertex. To find the x-intercepts, you can set $y = 0$, and then factor and solve for x.

vertex: $(2, -25)$ x-intercepts: $(7, 0)$, $(-3, 0)$ $\qquad\qquad$ vertex: $(4, -4)$ x-intercepts: $(6, 0)$, $(2, 0)$





12. Given $f(x) = -2x^2 + 16x - 34$, answer the following.

Does the function have a minimum or a maximum value? Maximum, because the negative coefficient of x^2 means it opens downward, making the vertex a maximum.

At what x value does the min/max occur? $x = \frac{-16}{2(-2)} = 4$ so $x = 4$.

What is the min/max value? $y = f(4) = -2(4^2) + 16(4) - 34$ so $y = -2$

13. A supply company manufactures copy machines. The unit cost C (cost in dollars to make each copy machine) depends on the number of machines made. If x machines are made, then the unit cost is given by $C(x) = 0.5x^2 - 170x + 25,850$. What is the minimum unit cost?

Since this is a quadratic equation, with a leading coefficient that is positive, it opens upward, making its vertex a minimum. To find the minimum unit cost, we need to find the y-value of the vertex.

$x = \frac{170}{2(0.5)} = 170 \quad C(170) = 0.5(170^2) - 170(170) + 25,850 = 11,400$ so the minimum is \$11, 400.

14. $s(x) = 3x + 6 \qquad t(x) = 4x \qquad u(x) = x^2 + 7 \qquad w(x) = \sqrt{x + 8}$

Given the functions defined above, find the following expressions.

$(s + t)(x)$

$= 3x + 6 + 4x = 7x + 6$

$(s \cdot t)(x)$

$= (3x + 6)(4x) = 12x^2 + 24x$

$(s - t)(4)$

$(s - t)(x) = -x + 6$ so $(s - t)(4) = -4 + 6 = 2$

$w(u(x))$

$= w(x^2 + 7) = \sqrt{x^2 + 7 + 8} = \sqrt{x^2 + 15}$

$w(u(1))$

$= \sqrt{1^2 + 15} = \sqrt{16} = 4$

$u(w(1))$

$u(\sqrt{1 + 8}) = u(3) = 3^2 + 7 = 16$

15. For each pair of functions below, find $f(g(x))$ and $g(f(x))$. Then determine whether f and g are inverses of each other.

$f(x) = \frac{6}{x}$ $\qquad$ $f(x) = 2x + 3$ $\qquad$ $f(x) = \frac{x+7}{5}$

$g(x) = \frac{6}{x}$ $\qquad$ $g(x) = 2x - 3$ $\qquad$ $g(x) = 5x - 7$

To show that two functions are inverses of one another, you need to show that $f(g(x)) = x$ as well as $g(f(x)) = x$.

$f(g(x)) = \frac{6}{\frac{6}{x}} = \frac{6}{1} \cdot \frac{x}{6} = x$ $\qquad$ $f(g(x)) = f(2x-3) = 2(2x-3)+3 = 4x-3 \neq x$ $\qquad$ $f(g(x)) = f(5x-7) = \frac{5x-7+7}{5} = x$

$g(f(x)) = \frac{6}{\frac{6}{x}} = \frac{6}{1} \cdot \frac{x}{6} = x$ $\qquad$ notice this is not an inverse from $f(g(x)) \neq x$. $\qquad$ $g(f(x)) = g(\frac{x+7}{5}) = \frac{5}{1}(\frac{x+7}{5}) - 7 = x$

16. h is a one-to-one function. Find $h^{-1}(x)$.

$h(x)4x + 3$ $\qquad$ $h(x) = 5x^3 + 7$ $\qquad$ $h(x) = \sqrt[3]{2x+5}$

There was a typo on the first problem. It should read $h(x) = 4x + 3$. To find an inverse function, you can (1) set $f(x) = y$, then (2) switch your x and y variables, and then (3) solve for y. This is now your $f^{-1}(x)$. I'm am going to start with step 2 being done here.

$x = 4y + 3$ solve for y, & $\qquad$ $x = 5y^3 + 7$ solve for y, & $\qquad$ $x = \sqrt[3]{2y+5}$ solve for y, &

$\frac{x-3}{4} = h^{-1}(x)$ $\qquad$ $\sqrt[3]{\frac{x-7}{5}} = h^{-1}(x)$ $\qquad$ $\frac{x^3-5}{2} = h^{-1}(x)$

17. f is a one-to-one function, $f(x) = \sqrt{x+5} + 4$.

Find the domain and range of $f(x)$. Then find $f^{-1}(x)$ and its domain.

Domain of f: $x \geq -5$ or you can write $[-5, \infty)$

Range of f: $y \geq 4$ or $[4, \infty)$ notice the lowest number the square root can be is zero and if we then add 4, the lowest y value possible for this function is 4.

To find the inverse: $x = \sqrt{y+5} + 4$ solve this for y, $(x-4)^2 - 5 = y$ so $f^{-1}(x) = (x-4)^2 - 5$

Remember that inverse functions swap domains and ranges. So the domain of f^{-1} is simply the Range of f, so $[4, \infty)$.