Operations with Radicals

Multiplication Property of Radicals: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$, where *a* and *b* are real numbers such that $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are also real numbers.

1. Multiply the radicals. Always simplify when you can.

 $\sqrt{5} \cdot \sqrt{5} \qquad \qquad \sqrt[3]{7} \cdot \sqrt[3]{3} \qquad \qquad \sqrt[3]{2} \cdot \sqrt[3]{12} \qquad \qquad \sqrt{5x^3} \cdot \sqrt{10x^4}$

 $(\sqrt{3ab^2})(\sqrt{21a^2b})$

 $\sqrt[3]{m^2n^2} \cdot \sqrt[3]{48m^4n^2}$

$$(4\sqrt{3xy^3})(-2\sqrt{6x^3y^2})$$

2. Notice that $\sqrt{5} \cdot \sqrt{5} = (\sqrt{5})^2 = 5$, and generally, $\sqrt{a} \cdot \sqrt{a} = (\sqrt{a})^2 = a$ when $a \ge 0$. We can even extend this to any real root, by saying, If $\sqrt[n]{a}$ is a real number, then $(\sqrt[n]{a})^n = a$. Using this idea, try the following, and again, always simplify when possible.

$$(\sqrt{3} + 2\sqrt{10})(4\sqrt{3} - \sqrt{10}) \qquad (4\sqrt{13xy})^2 \qquad (\sqrt{x-1})^2$$

Problem 2 continued.. $(-2\sqrt[3]{6wz^2})^3$

 $(\sqrt{p}-\sqrt{7})^2$

 $(\sqrt{5} + \sqrt{w})(\sqrt{5} - \sqrt{w})$

Multiplying Radicals with Different Indices. So far we have been multiplying radicals of the same type, like both being square roots, or both being cube roots, etc. Remember, we can write radicals as fractional exponents. We will use this idea to multiply radicals of different indices.

Example: $\sqrt[3]{5} \cdot \sqrt{5} = 5^{\frac{1}{3}} \cdot 5^{\frac{1}{2}} = 5^{\frac{2}{6}} \cdot 5^{\frac{3}{6}} = 5^{\frac{5}{6}} = \sqrt[6]{5^5}.$

Now you try:

$$\sqrt{x} \cdot \sqrt[4]{x} \qquad \qquad \sqrt[5]{q^4} \cdot \sqrt[3]{q^2} \qquad \qquad \frac{\sqrt{u^3}}{\sqrt[3]{u}} \qquad \qquad \frac{\sqrt{v^5}}{\sqrt[4]{v}}$$