Rationalizing the Denominator and Solving Equations with Radicals

1. Simplify the following.

$$\sqrt{\frac{4w^3}{25y^4}} \qquad \sqrt[5]{\frac{-32x}{y^{10}}} \qquad \frac{\sqrt{72ab^5}}{\sqrt{8ab}} \qquad \frac{\sqrt[3]{128wz^8}}{\sqrt[3]{2wz^2}}$$

Rationalizing the Denominator is the process of removing a radical from the denominator. To begin this process, we need to identify the missing radicand to be able to take the root.

Example: $\sqrt{a} \cdot \sqrt{?} = \sqrt{a^2} = a \qquad ? = a$

$\sqrt[3]{y} \cdot \sqrt[3]{?} = \sqrt[3]{y^3} = y \qquad ? = y^2$

$\sqrt[5]{2z^3} \cdot \sqrt[5]{?} = \sqrt[3]{2^5 z^5} = 2z$ find ?.

2. Using your answers from above, rationalize the denominators of the fractions below by multiplying both the numerator and denominator by the correct $\sqrt[n]{?}$ from above.

$$\frac{1}{\sqrt{a}} \qquad \frac{5}{\sqrt[3]{y}} \qquad \frac{4z^2}{\sqrt[5]{2z^3}}$$

Now Try These (Hint: It may help to write the constants in factored form):

$$\frac{b^3}{\sqrt{3}} \qquad \frac{2}{\sqrt{48}} \qquad \frac{1}{\sqrt[3]{9}} \qquad \sqrt[3]{\frac{5}{z^2}} \qquad \frac{6}{\sqrt[3]{3y^2}}$$

3. We've seen that $(a+b)(a-b) = a^2 - b^2$, so we will use that to rationalize the denominators with two terms. For example, use this to simplify $(2+\sqrt{6})(2-\sqrt{6})$.

Notice your answer no longer has square roots. Use this example to rationalize the denominator of $\frac{-2}{2+\sqrt{6}}$.

Now try these:

$$\frac{-12}{\sqrt{5}-3} \qquad \frac{x-5}{\sqrt{x}+\sqrt{5}} \qquad \frac{3\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \qquad \frac{3\sqrt{10}}{2+\sqrt{10}}$$

We know that $(\sqrt{x})^2 = x$ for real valued roots, and generally $(\sqrt[n]{x})^n = x$. We can use this to solve radical equations. For example, with $\sqrt[3]{x} = 4$, we can cube both sides to solve for x, $(\sqrt[3]{x})^3 = 4^3$ to get $x = 64$.

However, when we raise both sides to an even power, we might introduce false solutions (since raising to an even power can change the sign). For example, $\sqrt{x} = -7$, when we square both sides to solve for x, we get $x = 49$. Now if we check that solution by plugging $x = 49$ back into the original equation, we get $\sqrt{49} = -7$. This is incorrect, since the principal square root of x must be non-negative. Therefore, **we must check our solutions when raising both sides to an even power**.

4. Solve the following radical equations. please isolate the root first, before raising both sides to a power.

   $\sqrt{x} + 4 = 6$ $\qquad$ $\sqrt{5y+1} = 4$ $\qquad$ $(2z-3)^{\frac{1}{2}} - 3 = 6$

   $(k+18)^{\frac{1}{3}} + 5 = 3$ $\qquad$ $2\sqrt{3-w} - w = 0$ $\qquad$ $\sqrt{5y+1} + 2 = y + 3$

   $\sqrt[4]{3b+6} - \sqrt[4]{7b-6} = 0$ $\qquad$ $\sqrt{5a-9} = \sqrt{5a} - 3$

5. The time, $t(d)$, in seconds it takes for an object to drop d meters is given by $t(d) = \sqrt{\dfrac{d}{4.9}}$. Approximate the height of the Willis Tower in Chicago if it takes an object 9.51 seconds to drop from the top of the tower. Round to the nearest meter.