Complex Numbers

Complex Numbers Definition of i: $i = \sqrt{-1}$, which means that $i^2 = -1$.
$\sqrt{-b} = i\sqrt{b}$ for positive real number b.

1. Simplify the following.

   $\sqrt{-81}$ $\qquad$ $-\sqrt{-25}$ $\qquad$ $\sqrt{-50}$ $\qquad$ $-\sqrt{-20}$

2. When simplifying the product or quotient of an imaginary number, first simplify in terms of i, and then perform the multiplication or division.

   For Example, $\sqrt{-2} \cdot \sqrt{-18} = i\sqrt{2} \cdot 3i\sqrt{2} = 3i^2 \cdot (\sqrt{2})^2 = 3(-1) \cdot 2 = -6$. Now you try these:

   $\sqrt{-9} \cdot \sqrt{-16}$ $\qquad$ $\sqrt{-12} \cdot \sqrt{-50}$ $\qquad$ $\dfrac{\sqrt{-27}}{\sqrt{9}}$ $\qquad$ $\dfrac{\sqrt{-125}}{\sqrt{45}}$

3. **Powers of i**:

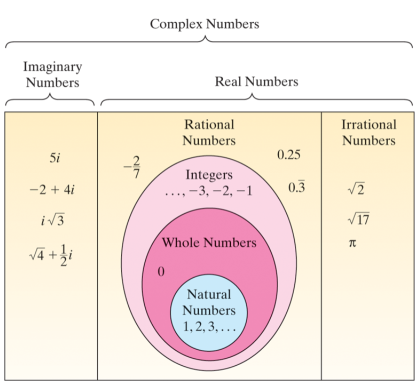
i^n	Decomposed form	Simplified form	i^n	Decomposed form	Simplified form
i	i^1	i	i^5	$i^4 \cdot i = 1 \cdot i$	i
i^2	$i \cdot i$	-1	i^6	$i^4 \cdot i^2 = 1 \cdot -1$	-1
i^3	$i^2 \cdot i = -1 \cdot i$	$-i$	i^7	$i^4 \cdot i^3 = 1 \cdot -i$	$-i$
i^4	$i^2 \cdot i^2 = -1 \cdot -1$	1	i^8	$i^4 \cdot i^4 = 1 \cdot 1$	1

   Do you see the pattern? Use that pattern to simplify the following:

   i^{13} $\qquad$ i^{26} $\qquad$ i^{43}

Definition of Complex Numbers: A complex number is a number of the form $a + bi$, where a and b are real numbers, and $i = \sqrt{-1}$.

If $b = 0$, the complex number is also a real number. If $b \neq 0$, it is an imaginary number. $a + bi$ and $a - bi$ are called complex conjugates.



4. Addition/Subtraction of complex numbers. $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$, in other words, like we have been doing for like terms, add/subtract the real parts and add/subtract the imaginary parts, keeping them separate. Try the following:

$(2 - i) + (5 + 7i)$ $\qquad$ $-\frac{7}{5}i - (-\frac{2}{5} + \frac{3}{5}i)$ $\qquad$ $(2 + 3i) - (1 - 4i) + (-2 + 7i)$

5. Multiplication of complex numbers.

$6i(1 - 3i)$ $\qquad$ $(2 - 10i)(3 + 2i)$ $\qquad$ $(4 + 5i)^2$ $\qquad$ $(5 + 2i)(5 - 2i)$

6. For division of complex numbers, we multiply the numerator and denominator of the fraction by the complex conjugate of the denominator. Try the following. Write your answer in $a + bi$ form.

$\frac{2}{1 + 3i}$ $\qquad$ $\frac{-i}{4 - 3i}$ $\qquad$ $\frac{7 + 3i}{4 - 2i}$ $\qquad$ $\frac{-6 - i}{-i}$

7. Simplify the following and write in $a + bi$ form.

$\frac{2 + \sqrt{-16}}{8}$ $\qquad$ $\frac{-6 + \sqrt{-72}}{6}$ $\qquad$ $\frac{-5 + \sqrt{-50}}{10}$