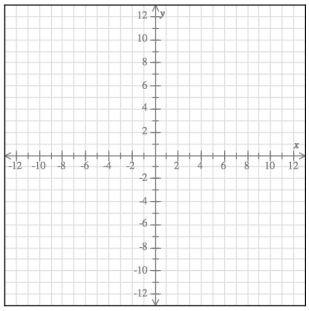
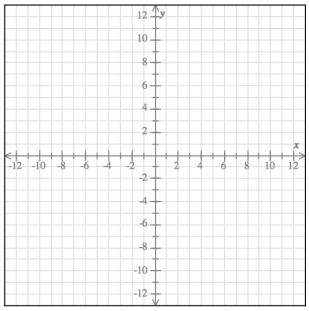
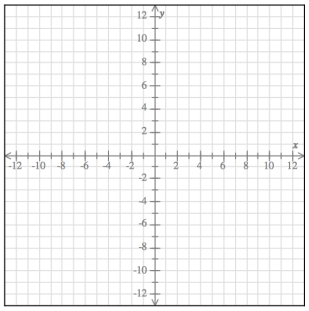
Graphs of Quadratic Equations

1. Sketch the following graphs by plotting points, and compare these to the graph of $y = x^2$.

   $f(x) = 2x^2$ $g(x) = -3x^2$ $h(x) = \frac{1}{2}x^2$

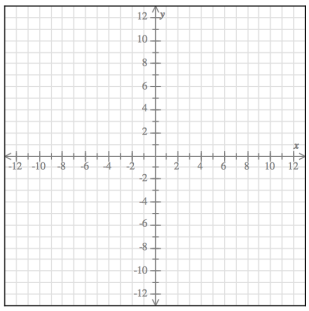
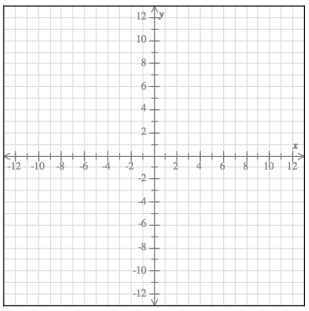
2. Generally, if $f(x) = ax^2$, describe how a affects the graph of $y = x^2$ if $0 < a < 1$, if $a > 1$, if $-1 < a < 0$, or if $a \leq -1$.

3. Putting this all together, we see that a quadratic equation of the form $f(x) = a(x - h)^2 + k$, which is called the **Vertex Form**:

   - Is the graph of $y = x^2$ shifted h spaces to the right or left (depending on the sign of h), and shifted k spaces up or down (depending on the sign of k). This means that its **vertex** is (h, k).
   - The axis of symmetry is $x = h$.
   - If $a > 0$, the parabola opens upward, and k is the minimum value of the function.
   - If $a < 0$, the parabola opens downward, and k is the maximum value of the function.

4. Graph the following. First state their vertex and axis of symmetry. Also, state whether the function has a minimum or a maximum and what that value is.

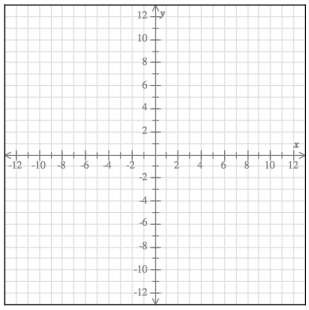
   $f(x) = (x + 1)^2 - 3$ $g(x) = -2(x - 2)^2 + 4$

Quadratic equations will not always be written in the vertex form. We can use completing the square to see that when a quadratic equation is in standard form, $ax^2 + bx + c$, the vertex is given by $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

5. For the following quadratic function, find the vertex, x and y-intercepts, and axis of symmetry. State whether it opens up or down and has a minimum or maximum. Where (at what x value) does the min/max occur, and what is that min/max value. Sketch a graph using what you have found.

   $f(x) = x^2 - 6x + 5$



6. Ben sells used iPhones. The average cost to package iPhones is given by the equation $C(x) = 3x^2 - 120x + 1300$, where x is the number of iPhones packaged per month.

   Determine the number of iPhones that Ben needs to package in a month to minimize the average cost.

   What is the minimum cost?

7. An arrow is shot straight upward into the air from the ground with an initial velocity of 128 $\frac{ft}{sec}$. The height of the arrow off the ground (in feet) is represented by $h(t) = -16t^2 + 128t$, where t is the number of seconds after it is shot. Answer the following.

   - At what time does the arrow reach its max height?
   - What is its max height?
   - When will it reach the ground again?