Composition of Functions and Functions and Inverses

$$f(x) = x + 4 \qquad g(x) = 2x^2 + 4x \qquad h(x) = x^2 + 1 \qquad k(x) = \frac{1}{x}$$

1. Given the functions above, find the following.

   $(f+g)(x)$ $\qquad$ $(g-f)(x)$ $\qquad$ $(g \cdot k)(x)$ $\qquad$ $\left(\frac{h}{k}\right)(x)$

   $(h \circ f)(x) = h(f(x))$ $\qquad$ $(k \circ h)(x) = k(h(x))$ $\qquad$ $(f \circ g)(x) = f(g(x))$

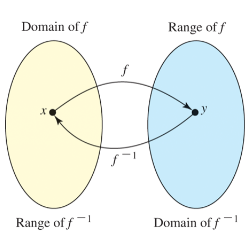
We defined a function by saying that it is a relation that assigns each x (domain) value exactly one y (range) value. We are now going to look at **Inverse Functions**. The Inverse Function, $f^{-1}(x)$ of a function $f(x)$ exchanges the domain and range of $f(x)$, meaning that the domain of $f(x)$ becomes the range of f^{-1}, and the range of $f(x)$ becomes the domain for $f^{-1}(x)$.

2. If the following points for a function $f(x)$ represents the pounds of coffee sold as x and the total for that coffee as y, $f = [(1, 8.50), (4, 34), (1.5, 12.75)]$. This means that 1 pound of coffee costs \$8.50, 4 pounds cost \$34, and 1.5 pounds costs \$12.75.

   Find f^{-1} and state the meaning of the points.

Since we interchange x and y values for the inverse function of a function, then for a function to have an inverse, if the x values in two coordinate pairs are different, then the y values must also be different. Remember we had the vertical line test to test if y is a function of x, now we will use the **Horizontal Line Test** to see if f is **one-to-one**, which is necessary for it to have an inverse.

Now if f maps x to y and f^{-1} maps y back to x, see picture below, then $f(f^{-1}(x)) = x = f^{-1}(f(x))$. So inverse functions undo each other and get us back to x, very handy!



3. Verify that f and g are inverses by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

   $f(x) = 6x + 1$ $\qquad$ $f(x) = \dfrac{\sqrt[3]{x}}{2}$

Since inverse functions exchange x and y values, to find an inverse function of a one-to-one function, $y = f(x)$, we can follow these steps:

- Replace $f(x)$ with y.
- Interchange x and y.
- Solve for y.
- Replace y with $f^{-1}(x)$.

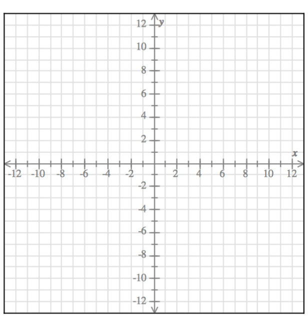
4. Use the steps above to find an equation for the inverse of each of the one-to-one functions below.

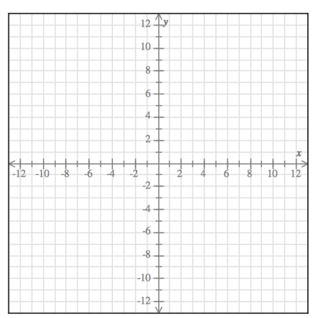
$f(x) = \frac{1}{3}x - 2$ $\qquad$ $n(x) = 4x + 2$ $\qquad$ $h(x) = \dfrac{4x - 1}{3}$

$g(x) = x^3 + 1$ $\qquad$ $k(x) = 4\sqrt[3]{x - 5}$

We use the horizontal line test to see if a function is one-to-one. If a function fails this test, we may be able to restrict its domain so that it is one-to-one.

5. Sketch a graph of $f(x) = x^2 + 3$. Does it pass the horizontal line test? If not, how can we restrict its domain (which x-values can we limit or function to) so that it is one-to-one? Sketch the new graph (restricted domain) on the second coordinate plane.





Label some of your points on your restricted domain, and since the inverse function will interchange x and y values of this function, plot points for the inverse function on the same coordinate plane and sketch its graph.

6. Now look back at problem 5. State the domain and range of both f and f^{-1}. Find an equation for $f^{-1}(x)$.