MATH 181 (WHYTE), SPRING 08. SAMPLE PROBLEMS

(1) Use the trapezoid rule with $n = 2$ to estimate the arc-length of the curve $y = \sin x$ between $x = 0$ and $x = \pi$.

(2) (a) Let R be the region between $y = \frac{1}{1+x^2}$ and the x-axis with $x \geq 0$. Does R have finite area? If so, what is the area?
(b) Let S be the solid obtained by revolving R around the y-axis. Does S have finite volume? If so, what is the volume?

(3) Evaluate the following integrals:
(a) $\int_{-\pi}^{\pi} \sin^4 x dx$
(b) $\int_0^1 \frac{dx}{2x^2+5x+2}$
(c) $\int_0^1 \frac{dx}{2x^2+4x+3}$
(d) $\int_0^\infty x^2 e^{-x} dx$

(4) Use a Taylor polynomial for $y = e^x$ to calculate e to two decimal places. Explain (using the remainder formula) why you have used sufficiently many terms.

(5) Let S be the surface obtained by revolving the curve $y = \sin x$ between $x = 0$ and $x = \pi$ around the x-axis. What is the surface area of S?

(6) (a) Estimate $\ln \frac{3}{2}$ using the degree two Taylor polynomial for $y = \ln x$ around $x = 1$.
(b) Estimate $\ln \frac{3}{2}$ using the Midpoint rule with $n = 2$ for the integral $\int_1^{\frac{3}{2}} \frac{dx}{x}$.
(c) Calculate the error bounds for the two estimates? Does this tell you which is closer to the exact answer?

(7) Does the improper integral $\int_0^\infty \frac{dx}{1+x^3}$ converge or diverge? Justify your answer.

(8) What is the arc-length of the segment of the parabola $y = 4 - x^2$ above the x-axis?

(9) Find a formula for the general Taylor polynomial $T_n(x)$ for the following functions around the specified points:
   (a) e^{-x^2} around $x = 0$
   (b) $\sqrt{x}$ around $x = 1$