1. To find an equation for the tangent plane we need a vector $\overrightarrow{\mathbf{n}}$ that is normal to the plane. This vector is found by taking the gradient of $F(x, y, z)$ and evaluating it at the given point $(1, -3, 2)$. The gradient is $\nabla F = \langle 8x, -2y, 6z\rangle$. Thus, the normal vector is $\overrightarrow{\mathbf{n}} = \nabla F(1, -3, 2) = \langle 8, 6, 12\rangle$ and an equation for the tangent plane is $8(x - 1) + 6(y + 3) + 12(z - 2) = 0$.

2. (a) The gradient of f is $\nabla f = \langle 2x - z + yz, xz, -x + xy\rangle$. At the point $(1, 1, 1)$ we have $\nabla f(1, 1, 1) = \langle 2, 1, 0\rangle$. The rate of change of f at $(1, 1, 1)$ in the direction of $\hat{\mathbf{v}}$ is $D_{\mathbf{v}}f(1, 1, 1) = \nabla f(1, 1, 1) \cdot \hat{\mathbf{v}} = \langle 2, 1, 0\rangle \cdot \frac{1}{\sqrt{6}}\langle 2, -1, 1\rangle = \frac{3}{\sqrt{6}}$.

   (b) The direction of most rapid increase of f at $(1, 1, 1)$ is $\hat{\mathbf{u}} = \dfrac{\nabla f}{||\nabla f||} = \dfrac{\langle 2, 1, 0\rangle}{\sqrt{5}}$. The maximum rate of change of f at the point is $||\nabla f|| = \sqrt{5}$.

   (c) When the position is $(1, 1, 1)$, we know that $t = 1$ since $\overrightarrow{\mathbf{p}}(1) = \langle 1, 1, 1\rangle$. The velocity is $\overrightarrow{\mathbf{p}}'(t) = \langle 1, 2t, 3t^2\rangle$. At $t = 1$ we have $\overrightarrow{\mathbf{p}}'(1) = \langle 1, 2, 3\rangle$. Thus, the rate of change of temperature f with respect to time t at $t = 1$ is $\left.\dfrac{df}{dt}\right|_{t=1} = \overrightarrow{\nabla} f(1, 1, 1) \cdot \overrightarrow{\mathbf{p}}'(1) = \langle 2, 1, 0\rangle \cdot \langle 1, 2, 3\rangle = 4$.

3. To find the critical points, we must solve the system of equations $f_x = 4x^3 + 4y = 0$, $f_y = 4y^3 + 4x = 0$. Solving the first equation for y we get $y = -x^3$. Plugging this into the second equation and simplifying gives us $x^9 - x = x(x^8 - 1) = 0$. There are only 3 real solutions: $x = 0$, $x = \pm 1$. Using $y = -x^3$ we find that $y = 0$ when $x = 0$, $y = -1$ when $x = 1$, and $y = 1$ when $x = -1$. Thus, the critical points are $(0, 0)$, $(1, -1)$, and $(-1, 1)$.

   To classify each point, we use the Second Derivative Test. The discriminant function $D(x, y)$ is $D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = (12x^2)(12y^2) - 4^2 = 144x^2y^2 - 16$. Its values at the critical points are $D(0, 0) = -16$, $D(1, -1) = 128$, and $D(-1, 1) = 128$. Since $D(0, 0) < 0$, the point $(0, 0)$ is a saddle point. Since $D(1, -1) > 0$ and $f_{xx}(1, -1) = 12 > 0$, the point $(1, -1)$ corresponds to a local minimum. Finally, since $D(-1, 1) > 0$ and $f_{xx}(-1, 1) = 12 > 0$, the point $(-1, 1)$ corresponds to a local minimum.

4. (a) Using the method of Lagrange multipliers, the system of equations that needs to be solve to determine the maximum value of f is

$$3x^2 = \lambda(2x), \quad 2y = \lambda(6y), \quad -3z^2 = \lambda(4z), \quad x^2 + 3y^2 + 2z^2 = 3$$

(b) Given the list of solutions to the above equations, the maximum value of f is found by evaluating f at the solutions and choosing the largest value.

5. Changing the order of integration and evaluating gives us

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dy\, dx = \int_0^1 \int_0^{3y^2} e^{y^3}\, dx\, dy = \int_0^1 3y^2 e^{y^3}\, dy = e^{y^3}\Big|_0^1 = e - 1$$

6. To find the surface area we use the parametrization $\Phi(u, v) = (u\cos v, u\sin v, -u^2)$ where $0 \le u \le \sqrt{20}$ and $0 \le v \le 2\pi$. The tangent vectors $\overrightarrow{\mathbf{T}}_u$ and $\overrightarrow{\mathbf{T}}_v$ are

$$\overrightarrow{\mathbf{T}}_u = \frac{\partial \Phi}{\partial u} = \langle \cos v, \sin v, -2u \rangle, \quad \overrightarrow{\mathbf{T}}_v = \frac{\partial \Phi}{\partial v} = \langle -u\sin v, u\cos v, 0 \rangle$$

The normal vector is $\overrightarrow{\mathbf{n}}(u, v) = \overrightarrow{\mathbf{T}}_u \times \overrightarrow{\mathbf{T}}_v = \langle 2u^2\cos v, 2u^2 \sin v, u \rangle$ and its magnitude is $||\overrightarrow{\mathbf{n}}(u, v)|| = \sqrt{4u^4 + u^2} = u\sqrt{4u^2 + 1}$. The surface area is then

$$\text{Surface Area} = \iint_{\mathcal{S}} ||\overrightarrow{\mathbf{n}}(u, v)|| du\, dv = \int_0^{2\pi} \int_0^{\sqrt{20}} u\sqrt{4u^2+1}\, du\, dv = \frac{364\pi}{3}$$