1. The partial derivatives are \( f_x = 6x + y \) and \( f_y = x + 4y \). At the point \((1, 1)\) we have \( f_x(1, 1) = 7, \ f_y(1, 1) = 5 \). The linearization is \( L(x, y) = 6 + 7(x - 1) + 5(y - 1) \). Then, \( f(1.1, 1.2) \approx L(1.1, 1.2) = 7.7 \).

2. We must solve the equations \( f_x = 3x^2 - 3y = 0 \) and \( f_y = -3x + 3y^2 = 0 \). The solutions are \((0, 0)\) and \((1, 1)\). The discriminant at \((0, 0)\) is \( D(0, 0) = -9 \) and the discriminant at \((1, 1)\) is \( D(1, 1) = 27 \). Furthermore, \( f_{xx}(1, 1) = 6 \). Therefore, \( f \) has a saddle at \((0, 0)\) and a local minimum at \((1, 1)\).

3. To solve the double integral, it is necessary to change the order of integration:

\[
\int_0^2 \int_0^{x^2} \sin \left(x^3\right) \, dy \, dx = \int_0^2 x^2 \sin \left(x^3\right) \, dx = \left[-\frac{1}{3} \cos \left(x^3\right)\right]_0^2 = \frac{1}{3} - \frac{1}{3} \cos 8.
\]

4. Using the method of Lagrange multipliers, we must solve the equations:

\[
1 = \lambda \left(\frac{1}{2}x\right), \quad 1 = \lambda \left(\frac{2}{9}y\right), \quad -1 = \lambda(2z), \quad \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1
\]

Solving the first three equations for \( x, y, \) and \( z \), respectively, and then plugging into the last equation we get \( \lambda = \pm \sqrt{\frac{7}{2}} \). Substituting each value of \( \lambda \) back into the first three equations, we get the points

\[
P_1 = \left(2 \sqrt[7]{\frac{2}{9}}, \ 2 \sqrt[7]{\frac{2}{7}}, \ -1 \sqrt[7]{\frac{2}{7}}\right), \ P_2 = \left(-2 \sqrt[7]{\frac{2}{7}}, \ -9 \sqrt[7]{\frac{2}{7}}, \ 1 \sqrt[7]{\frac{2}{7}}\right).
\]

The minimum value of \( f \) is \( -\sqrt{14} \) at \( P_2 \) and the maximum value of \( f \) is \( \sqrt{14} \) at \( P_1 \).

5. Let \( F(x, y, z) = x^2 + y^3 - 2z \). The gradient is \( \nabla F = (2x, 3y^2, -2) \). At the point \((1, 2, 4)\) we have \( \nabla F(1, 2, 4) = (2, 12, -2) \). The equation for the tangent plane is \( 2(x - 1) + 12(y - 2) - 2(z - 4) = 0 \).
6. The gradient is $\nabla F = (6x, 2y, -8z)$. At the point $(1, -4, 3)$ we have $\nabla F(1, -4, 3) = (6, -8, -24)$. The equation for the tangent plane is $6(x - 1) - 8(y + 4) - 24(z - 3) = 0$.

7. We must solve the equations $f_x = x^2 - y = 0$ and $f_y = 2y - x = 0$. The solutions are $(0, 0)$ and $(\frac{1}{2}, \frac{1}{4})$. The discriminant at $(0, 0)$ is $D(0, 0) = -1$ and the discriminant at $(\frac{1}{2}, \frac{1}{4})$ is $D(\frac{1}{2}, \frac{1}{4}) = 1$. Furthermore, $f_{xx}(\frac{1}{2}, \frac{1}{4}) = 1$. Therefore, $f$ has a saddle at $(0, 0)$ and a local minimum at $(\frac{1}{2}, \frac{1}{4})$.

8. Using the method of Lagrange multipliers, we must solve the equations:

$$2x = \lambda(2x), \quad -1 = \lambda(2y), \quad x^2 + y^2 = 4$$

The first equation tells us that $x = 0$ or $\lambda = 1$. If $x = 0$, then the third equation tells us that $y = \pm 2$. If $\lambda = 1$, then the second equation tells us that $y = -\frac{1}{2}$ and the third equation tells us that $x = \pm \frac{\sqrt{15}}{2}$. Therefore, we must evaluate $f$ at the four points

$$P_1 = (0, 2), \quad P_2 = (0, -2), \quad P_3 = \left(\frac{\sqrt{15}}{2}, -\frac{1}{2}\right), \quad P_4 = \left(-\frac{\sqrt{15}}{2}, -\frac{1}{2}\right)$$

The minimum value of $f$ is $-1$ at $P_1$ and the maximum value of $f$ is $\frac{17}{4}$ at $P_3$ and $P_4$.

9. The projection of the region onto the $xy$-plane is the portion of the circle of radius 1 centered at $(0, 0)$ in the first quadrant. Therefore, the volume is:

$$V = \int \int_D (1 - x^2 - y^2) \, dA = \int_0^{\pi/2} \int_0^1 (1 - r^2) r \, dr \, d\theta = \frac{\pi}{8}$$

10. Using the method of Lagrange multipliers, we must solve the equations:

$$2x = \lambda(2x), \quad -2y = \lambda(2y), \quad 4z = \lambda(2z), \quad x^2 + y^2 + z^2 = 1$$

There are 6 sets of solutions to these equations:

$$P_1 = (0, 0, 1), \quad P_2 = (0, 0, -1), \quad P_3 = (0, 1, 0)$$
$$P_4 = (0, -1, 0), \quad P_5 = (1, 0, 0), \quad P_6 = (-1, 0, 0)$$

The minimum value of $f$ is $-1$ at $P_3$ and $P_4$ and the maximum value of $f$ is 2 at $P_1$ and $P_2$. 
11. The region is bounded below by $z = 0$ and above by $z = y = r \sin \theta$. The projection of $W$ onto the $xy$-plane is the upper half of the circle of radius 2 centered at $(0, 0)$. Therefore, the integral is:

$$
\iiint_{W} (x^2 + y^2)^{1/2} \, dV = \int_{0}^{\pi} \int_{0}^{2} \int_{0}^{r \sin \theta} r \cdot r \, dz \, dr \, d\theta = 8
$$

12. The region is bounded below by $z = -\sqrt{1 - r^2}$ and above by $z = \sqrt{1 - r^2}$. The projection of $B$ onto the $xy$-plane is the circle of radius 1 centered at $(0, 0)$. Therefore, the integral is:

$$
\iiint_{B} x^2 \, dV = \int_{0}^{2\pi} \int_{0}^{1} \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} (r \cos \theta)^2 \cdot r \, dz \, dr \, d\theta = \frac{4\pi}{15}
$$

13. The projection of the region onto the $xy$-plane is the circle of radius 1 centered at $(0, 0)$. Therefore, the volume is:

$$
V = \iint_{\mathcal{D}} [(2 - r^2) - r^2] \, dA = \int_{0}^{2\pi} \int_{0}^{1} (2 - 2r^2) \cdot r \, dr \, d\theta = \pi
$$

14. The first partial derivatives of $f$ are $\frac{\partial f}{\partial x} = ye^{xy}$ and $\frac{\partial f}{\partial y} = xe^{xy}$. The equations for polar coordinates are $x = r \cos \theta$ and $y = r \sin \theta$. Therefore, $\frac{\partial x}{\partial r} = \cos \theta$ and $\frac{\partial y}{\partial r} = \sin \theta$. Therefore, the partial derivative $\frac{\partial f}{\partial r}$ is:

$$
\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r} = (2r \sin \theta \cos \theta) e^{r^2 \sin^2 \theta} = \frac{2xy}{\sqrt{x^2 + y^2}} e^{xy}
$$

15. The projection of $A$ onto the $xy$-plane is the portion of the circle of radius 1 centered at $(0, 0)$ in the first quadrant. Using polar coordinates, the average value of $f$ on $A$ is:

$$
\bar{f} = \frac{\iint_{A} f(x, y) \, dA}{\iint_{A} dA} = \frac{\int_{0}^{\pi/2} \int_{0}^{1} (2 + r \cos \theta - r \sin \theta) \cdot r \, dr \, d\theta}{\int_{0}^{\pi/2} \int_{0}^{1} r \, dr \, d\theta} = 2
$$

16. The region of integration is a triangle bounded by the lines $y = 0$, $y = x$, and $x + y = 2$. 
The integral is then:
\[
\int \int_D \frac{x}{y+1} dA = \int_0^1 \int_y^{2-y} \frac{x}{y+1} dx \, dy = 4 \ln 2 - 2
\]

17. (a) \( f_x = 2x - 1 = 0, \quad f_y = 2y = 0 \quad \implies \quad x = \frac{1}{2}, \quad y = 0 \)
(c) On the boundary \( x = 0 \), the function becomes \( f(0, y) = y^2 \) and attains a maximum value of \( 1 \) at \( y = \pm 1 \) and a minimum value of \( 0 \) at \( y = 0 \). On the boundary \( x = 1 - y^2 \), the function becomes \( f(1 - y^2, y) = (1 - y^2)^2 - (1 - y^2) + y^2 = y^4 \) and attains a maximum value of \( 1 \) at \( y = \pm 1 \) and a minimum value of \( 0 \) at \( y = 0 \). Therefore, we have:
\[
f \left( \frac{1}{2}, 0 \right) = -\frac{1}{4}, \quad f(0, 0) = 0, \quad f(0, \pm 1) = 1, \quad f(1, 0) = 0
\]
The minimum value of \( f \) is \(-\frac{1}{4}\) at \((\frac{1}{2}, 0)\) and the maximum value of \( f \) is \( 1 \) at \((0, \pm 1)\).

18. The gradient of \( F \) is \( \vec{\nabla} F = \left< 2x + 4x^2e^{4x-y^2}, -2xe^{4x-y^2} \right> \). At the point \((1, 2)\) we have \( \vec{\nabla} F(1, 2) = \langle 6, -4 \rangle \). The direction of fastest growth is:
\[
\hat{u} = \frac{\vec{\nabla} F(1, 2)}{||\vec{\nabla} F(1, 2)||} = \left< \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right>
\]

19. The gradient of \( f \) is \( \vec{\nabla} f = \langle 2xy - e^{x+y}, x^2 - e^{x+y} \rangle \). The derivative of \( \vec{r}(t) \) is \( \vec{r}'(t) = \langle -e^{-t}, -\sin t \rangle \). At \( t = 0 \) we have \( \vec{r}(0) = \langle 1, 1 \rangle \implies x = 1, \quad y = 1 \) and \( \vec{r}'(0) = \langle -1, 0 \rangle \). Using the Chain Rule, we have:
\[
\left. \frac{d}{dt} f(\vec{r}(t)) \right|_{t=0} = \vec{\nabla} f(1, 1) \cdot \vec{r}'(0) = \left< 2 - e^2, 1 - e^2 \right> \cdot \left< -1, 0 \right> = e^2 - 2
\]

20. In spherical coordinates, the equation for sphere is \( \rho = 1 \) and the equation for the cone is \( \phi = \frac{\pi}{4} \). Therefore, the mass is:
\[
\text{mass} = \int \int \int_A f(x, y, z) \, dV = \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho \cdot \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{\pi}{2} \left( 1 - \frac{\sqrt{2}}{2} \right)
\]