MATH 442, MIDTERM REVIEW PROBLEMS

(1) Let c be the helix defined by $c(t) = (A\cos t, A\sin t, Bt)$ for some real numbers A and B. Compute the curvature and torsion of c.

(2) Let $c(t)$ be a parametrized curve in $\mathbb{R}^3$, not necessarily unit speed.
  (a) Find a formula for the curvature of c as a function of $c'(t)$ and $c''(t)$. Is this always defined? Why or why not?
  (b) Find a formula for the torsion of c as a function of $c'(t)$, $c''(t)$, and $c'''(t)$. Is this always defined? Why or why not?

(3) Let S be a regular surface in $\mathbb{R}^3$ such that $0 \notin S$. Define a map from S to the unit sphere by sending $p \in S$ to the point $\frac{p}{||p||}$.
  (a) Show that $p \in S$ is a critical point for this map if and only if the line from p to the origin is tangent to S at p.
  (b) Show that if S is the boundary of a convex set containing 0 then S is diffeomorphic to the sphere.

(4) Let c be a regular curve in $\mathbb{R}^2$ and let S (called the cylinder over c) be the set of points (x, y, z) such that $(x, y) \in c$. Show that S is a regular surface.

(5) Stereographic projection is the map from the 2-sphere $x^2+y^2+z^2 = 1$ with the point $(0, 0, -1)$ removed to $\mathbb{R}^2$ which is defined by sending a point p on the sphere to (x, y) such that the line through $(x, y, 1)$ and $(0, 0, -1)$ passes through p.
  (a) Show that this map is a diffeomorpshism
  (b) Show that the derivative at every point is conformal, meaning that it does not change the angles between vectors.

(6) Define explicitly a Mobius band as a subset of $\mathbb{R}^3$.
  (a) Prove the set you have defined is a regular surface.
  (b) Prove it is non-orientable.
  (c) Compute its area.