## MATH 530 (WHYTE), SPRING 08. SAMPLE PROBLEMS

(1) Prove that if $f$ is continuous but not differentiable at $x=a$ and $g$ is differentiable at $x=a$ then their product $f g$ is differentiable at $x=a$ iff $g(a)=0$.
(2) Prove directly from the definition that $x^{2}$ is differentiable everywhere.
(3) Using any of the product, sum, and chain rules (give the precise statements you use) prove that all polynomial functions are differentiable everywhere (hint: induction).
(4) (a) Prove that any twice differentiable function satisfying $f^{\prime \prime}+f=0$ is of the form $A \sin x+B \cos x$ for some constants $A$ and $B$ (alternative hint: consider the quantity $f(x)^{2}+f^{\prime}(x)^{2}$ ).
(b) Show that $f(x)=\sin (x+a)$ satisfies this differential equation, and from this prove the formula $\sin (x+a)=\sin x \cos a+$ $\sin a \cos x$.
(5) Prove that any bounded function which is discontinuous at only finitely many points is integrable.
(6) Let $f$ be the function on $[0,1]$ defined by:

- $f\left(\frac{1}{n}\right)=1$ for positive integers $n$.
- $f(x)=0$ for all other $x$

Where is $f$ continuous? Is $f$ integrable? Prove your answers correct.

