MATH 530 (WHYTE), SPRING 08. SAMPLE PROBLEMS

- (1) Prove that if f is continuous but not differentiable at x = a and g is differentiable at x = a then their product fg is differentiable at x = a iff g(a) = 0.
- (2) Prove directly from the definition that x^2 is differentiable everywhere.
- (3) Using any of the product, sum, and chain rules (give the precise statements you use) prove that all polynomial functions are differentiable everywhere (hint: induction).
- (4) (a) Prove that any twice differentiable function satisfying f'' + f = 0is of the form $A \sin x + B \cos x$ for some constants A and B (alternative hint: consider the quantity $f(x)^2 + f'(x)^2$).
 - (b) Show that $f(x) = \sin(x + a)$ satisfies this differential equation, and from this prove the formula $\sin(x + a) = \sin x \cos a + \sin a \cos x$.
- (5) Prove that any bounded function which is discontinuous at only finitely many points is integrable.
- (6) Let f be the function on [0, 1] defined by :
 - $f(\frac{1}{n}) = 1$ for positive integers n.
 - f(x) = 0 for all other x

Where is f continuous? Is f integrable? Prove your answers correct.