MATH 215, FALL 2018 (WHYTE) MIDTERM (OCT 5)

(1) Basic set operations

    (a) Decide which of the following statements are true and which are false :

        (i) $(A \cap B) \cup C = A \cap (B \cup C)$

        (ii) $(A^c \cup B^c) = (A \cup B)^c$

        (iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

        (iv) $A \cup B^c = (A \cap B) \cup (A^c \cap B^c)$

    (b) Choose one of the above and prove it or give a counter-example:

(2) Functions

    (a) Let $f : A \to B$ and $g : B \to C$ be functions. Decide which of the following statements are true and which are false :

        (i) If f and g are surjective then $g \circ f$ is surjective

        (ii) If $g \circ f$ is surjective then f is surjective

        (iii) If $g \circ f$ is surjective then g is surjective

    (b) Choose one of the above and prove it or give a counter-example:

(3) Images and Inverse images

    (a) Let $f : X \to Y$ be a function. Decide which of the following statements are true and which are false :

        (i) If $B \subset Y$ then $f^{-1}(B)^c = f^{-1}(B^c)$

        (ii) If $A \subset X$ then $f(A)^c = f(A^c)$

        (iii) If A_1 and A_2 are subsets of X then $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

        (iv) If B_1 and B_2 are subsets of Y then $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

    (b) Choose one of the above and prove it or give a counter-example: