MATH 215, FALL 2018 (WHYTE) SAMPLE SECOND MIDTERM

(1) Decide whether each of the following is true and either prove it or give a counter-example:

   (a) If $a|b$ then $a^2|b^2$
   (b) For all $n \in \mathbb{N}$ there is an $m \in \mathbb{N}$ with $n \leq m^2 \leq 2n$
   (c) If $a \equiv b \bmod n$ then $a^2 \equiv b^2 \bmod n^2$
   (d) For all n the greatest common divisor of n and $n+2$ is 1 or 2
   (e) There is no integer n with $n^2 = 10$

(2) What is wrong with the following proof?

   Claim: Every natural number is odd (meaning equal to $2k+1$ for some k).

   Proof: Let S be the set of natural numbers which are not odd. We need to show that S is empty. We will argue by contradiction, so assume S is non-empty. The well ordering principle then says that S has a smallest element, call it n.

   Consider $n-2$. Since 1 is odd, $1 \notin S$ so $n > 1$ and so $n-2 > 0$. Since $n-2 < n$ and n is the smallest element of S, we must have $n-2 \notin S$. This means that $n-2$ is odd, so $n-2 = 2k+1$ for some k. Then $n = n-2+2 = 2k+1+2 = 2(k+1)+1$ so n is also odd. This is a contradiction since $n \in S$. $\square$