MATH 215 (WHYTE, F18) SAMPLE FINAL

(1) Prove that if a and b are integers with $ab = 0$ then $a = 0$ or $b = 0$

(2) Prove that if p is a prime number and p divides a product $n_1 n_2 \dots n_k$ of integers then $p | n_i$ for some i

(3) Show that if n is odd and $2^{m_1} \equiv 2^{m_2} \mod n$ then $2^{m_2 - m_1} \equiv 1 \mod n$

(4) If $f : A \to B$ is an injective function then for all $S \subset A$, $S = f^{-1}(f(S))$. Give an example to show this isn't true without the assumption of injectivity.

(5) Prove that there is no integer n with $n^2 + 4n + 2 = 0$

(6) Show that if $a \equiv b \mod n$ then $a^k \equiv b^k \mod n$ for all $k \in \mathbb{N}$

(7) Suppose $f : A \to B$ and $g : B \to C$ are functions where g is injective and f is not surjective. Prove that $g \circ f$ cannot be surjective.

(8) Suppose $a \equiv b \mod n$, show that $gcd(a, n) = gcd(b, n)$