ESP Math 179 | **WORKSHEET 19** | Spring 2020

Worksheet 19

L'Hopital's rule

2 April 2020

---

1. **(Warm-up)** Find the derivative of $f(x) = e^{\sin(x)}$

2. **(Warm-up)** Find the derivative of $g(x) = \dfrac{x^2 + 3x}{\cos(x)}$

   L'Hôpital's rule says that if f and g are differentiable on an open interval containing a, with $g'(x) \neq 0$ except for possibly at a, and $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ (or $\pm\infty$), then

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

   if the second limit exists.

3. Evaluate the limit $\lim_{x\to 0} \dfrac{\sin(x)}{x}$ using L'Hôpital's rule.

4. Evaluate the following limits, using L'Hôpital's rule **only when necessary**.

   (a) $\lim_{t\to\infty} \dfrac{\ln(3t)}{t^2}$

   (b) $\lim_{x\to 3^+} \dfrac{x^2 - 3x + 1}{x - 3}$

   (c) $\lim_{y\to -4} \dfrac{\sin(\pi y)}{y + 4}$

   (d) $\lim_{z\to 0^+} \ln(z)\sin(z)$

   (e) **(Challenge)** $\lim_{w\to 0} \dfrac{\sin(x^4)}{\sin^4(x)}$

   (hint: L'Hôpital's rule works here, but it gets very complicated. See if there's a way to rewrite the function to make the limit easier to find, without using L'Hôpital.)

5. **(Group 1)** Try to use L'Hôpital's rule to evaluate $\lim_{x\to 0^+} \dfrac{\ln(x)}{1/x}$, **without** simplifying the function at each stage. What happens? What does this tell us about using L'Hôpital's rule?

6. **(Group 2)** Try to use L'Hôpital's rule to evaluate $\lim_{x\to\infty} \dfrac{e^x}{e^x + e^{-x}}$. What happens? What does this say about L'Hôpital's rule in this case?

7. **(Challenge)** Prove this simple case of L'Hôpital's rule: If $f(x), g(x)$ are differentiable on an open interval containing a, with $f(a) = g(a) = 0$ and $g'(a) \neq 0$, then

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$