ESP Math 179

Worksheet 18

Review of Limits & Optimization

31 March 2020

Spring 2020

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1. **(Warm-up)** Evaluate $\lim_{x\to 2^-} \dfrac{x+5}{x-2}$ and $\lim_{x\to 2^+} \dfrac{x+5}{x-2}$
2. **(Warm-up)** Find two positive numbers x and y that add up to 300 such that the product xy is maximized.

Group 1 Problems

3. Evaluate $\lim_{x\to 4} \dfrac{x^2+x-20}{x^2-6x+8}$
4. Evaluate $\lim_{x\to -\infty} \dfrac{3x^3+4x^2-20x+1}{x^3+100x-8}$. Be sure to show all work.
5. We wish to construct a box whose base length is 3 times its base width. The material used for the top and bottom costs \$0.20/cm^2 and the material used for the sides costs \$0.05/m^2. If the box must have a volume of 1000 cm^3, what are the dimensions of the box that minimize the cost?

Group 2 Problems

6. Evaluate $\lim_{x\to -3} \dfrac{x^2+3x-18}{x^2-2x-3}$
7. Evaluate $\lim_{x\to \infty} \dfrac{x^2-4x+1}{5x^3+12x^2-6x+10}$. Be sure to show all work.
8. We wish to construct a cylinder by rolling a rectangular sheet of paper into a tube. If the sheet of paper needs to have a perimeter of 32 inches, what are the dimensions of the paper that give the largest possible volume of the cylinder?

Bonus

Limits where direct substitution gives $\frac{0}{0}$ and $\frac{\infty}{\infty}$ are said to be in **indeterminate form**. We've previously seen some methods for computing these limits in certain cases. In 180, you'll soon be covering a more general technique known as **L'Hôpital's rule**. L'Hôpital's rule says that

$$\text{if } \frac{f(x)}{g(x)} \to \frac{0}{0} \text{ or } \frac{f(x)}{g(x)} \to \frac{\infty}{\infty} \text{ and } \lim_{x\to a} \frac{f'(x)}{g'(x)} \text{ exists,}$$
$$\text{then } \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

In other words, for limits in indeterminate form, we can use $\lim_{x\to a} \frac{f'(x)}{g'(x)}$, if it exists, to find the limit $\lim_{x\to a} \frac{f(x)}{g(x)}$ (note that L'Hôpital's rule does NOT work for limits not in indeterminate form).

9. Using L'Hôpital's rule, compute $\lim_{x\to -4} \dfrac{\sin(\pi x)}{x^2-16}$.