ESP Math 179 | Spring 2020

Worksheet 23

Definite Integrals

28 January 2020

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1. **(Review)** Evaluate the following limits:
   (a) $\lim_{x\to 0} \dfrac{\sin(6x)}{\sin(11x)}$
   (b) $\lim_{y\to 0^+} (e^y + y)^{\frac{1}{y}}$

1. Sigma Notation

2. Express the following sums using sigma notation
   (a) $1 + 2 + 4 + 8 + 16 + 32$
   (b) $0 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6}$
3. Use sigma notation to write the n-th right Riemann sum of a function $f(x)$ on the interval $[a, b]$.
4. Based on your formula from the above question, write down the 10th right Riemann sum of $f(x) = x^2 + x$ on the interval $[0, 5]$. Do not simplify.

2. Definite Integrals

5. Suppose that $\displaystyle\int_{-2}^{5} f(x)\,dx = 6$ and $\displaystyle\int_{-2}^{0} f(x)\,dx = -1$. Find the values of
   (a) $\displaystyle\int_{0}^{5} f(x)\,dx$
   (b) $\displaystyle\int_{5}^{-2} 2f(x)\,dx$
   (c) $\displaystyle\int_{-2}^{0} (3f(x) + 1)\,dx$
   (d) $\displaystyle\int_{0}^{0} 2xf(x)\,dx$
6. Sketch a graph of $f(x) = 2 - |x|$. Use this to compute $\displaystyle\int_{0}^{3} f(x)\,dx$.
7. The area of a rectangle is a function of two variables (i.e., it depends on two variables): $A(b, h) = b \cdot h$.
   (a) Explain the meaning of the equation $A(b, 2h) = 2A(b, h)$.
   (b) How is this related to the fact that $\displaystyle\int_{a}^{b} 2f(x)\,dx = 2\int_{a}^{b} f(x)\,dx$?