ESP Math 179 | Spring 2020

Worksheet 25

The Fundamental Theorem of Calculus

23 April 2020

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1. **(Review)** Find the linear approximation to $f(x) = e^x$ at $x = 0$, and use this to approximate the value of $e^{0.1}$.

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2. Consider the area under the graph of f between a fixed point a and a point x (which will vary). This area depends on the value of x, so we may say that the area $A(x)$ is a function of x. If we increase the value of x by a small amount dx, approximately how much will the area $A(x)$ change by?

3. What does this tell us about the value of $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt$?

   This leads us to the **Fundamental Theorem of Calculus, part 1**:

   Theorem. *Suppose f is continuous on $[a, b]$. If x is in (a, b), then*

   $$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

   Note that this means that $\int_a^x f(t)\,dt$ is an antiderivative of $f(x)$.

4. Evaluate the following:

   (a) $\dfrac{d}{dx}\displaystyle\int_3^x \frac{\sin(e^t) + 3t}{t\ln(\cos(t))}\,dt$

   (b) $\dfrac{d}{dz}\displaystyle\int_z^5 xe^{5x}\,dx$

   (c) $\dfrac{d}{dx}\displaystyle\int_0^{x^2} 3t\sin(t)\,dt$

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5. We want to be able to use the FTC to help us calculate integrals. Suppose f is a continuous function, and say we want to find $\displaystyle\int_a^b f(t)\,dt$. We will approach this in small steps:

   (a) Let $A(x) = \displaystyle\int_a^x f(t)\,dt$. For some fixed real number b, what does $A(b)$ represent? (recall what we said the integral represents)

   (b) Now let $F(x)$ be any antiderivative of $f(x)$. What can you say about how $F(x)$ and $A(x)$ are related? (hint: they are both antiderivatives of $f(x)$)

   (c) By letting $x = a$, can we say something more specific about our answer to part (b)?

   (d) Based on what we have found so far, what is a simple way for us to calculate $\displaystyle\int_a^b f(t)\,dt$? (hint: try plugging in $x = b$ into $A(x)$)?

   Our solution to problem 5 is sometimes referred to as the **Evaluation Theorem** or the **fundamental Theorem of Calculus, part 2**, and gives us a relatively easy way to evaluate integrals.

   Theorem. *Suppose f is continuous on $[a, b]$. If F is an antiderivative of f, then*

   $$\int_a^b f(x)\,dx = F(b) - F(a)$$

6. Use the Fundamental Theorem of Calculus, part 2 to evaluate the following integrals:

   (a) $\int_0^1 x + 1 \, dx$

   (b) $\int_0^{\frac{3\pi}{2}} \cos(t) \, dt$

   (c) $\int_2^4 3x^2 + 4x + 2 \, dx$