ESP Math 179

Worksheet 24

More Definite Integrals

21 April 2020

Spring 2020

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1. **(Review)** We are given 45 m^2 of cardboard and need to build a box with a square base and no top. What are the dimensions of the box that maximize its volume?

1. Summations

2. **(Warm-up)** Express the sum $2 + 5 + 8 + 11 + 14 + 17$ using sigma notation.

   For the following problem, it might be helpful to recall the following formulas:

$$\sum_{i=1}^{n} c = c \cdot n, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Evaluate the following sums:

   (a) $\displaystyle\sum_{i=1}^{n} \frac{15}{n}$

   (b) $\displaystyle\sum_{i=1}^{n} \frac{18i}{n^2}$

   (c) $\displaystyle\sum_{i=1}^{n} \frac{6i^2}{n^3}$

   (d) $\displaystyle\sum_{i=1}^{n} \frac{6i}{n^2}$

2. Limit Definition of the Integral

Recall the limit definition of the integral:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where Δx is the length of each subinterval, and x_i are the evaluation points (we'll use right endpoints).

4. Use the limit definition of the integral to evaluate $\displaystyle\int_1^4 2x + 3\,dx$. It may be helpful to break down the steps as follows:

   (a) If we divide the interval $[1, 4]$ into n subintervals of length Δx, what will be the values of Δx and x_i (the right endpoint of the i-th interval)?

   (b) What is $f(x_i)$?

   (c) For a fixed value of n, what is $\displaystyle\sum_{i=1}^{n} f(x_i)\Delta x$? The sums you did in Q3 might come in handy.

   (d) Evaluate $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$.

5. Use the limit definition of the integral to evaluate $\displaystyle\int_0^1 6x^2 - 6x\,dx$.

3. Sneak Peek

6. (a) Consider the area under the graph of f between a fixed point a and a point x (which will vary). This area depends on the value of x, so we may say that the area $A(x)$ is a function of x. If we increase the value of x by a small amount dx, approximately how much will the area $A(x)$ change by?

   (b) What does this tell us about the value of $\dfrac{d}{dx}\displaystyle\int_a^x f(x)\,dx$?