(1) (Warm-up) Determine if the following limits exist, and evaluate them if they do.
(a) $\lim _{x \rightarrow 2} \ln (x-2)$
(c) $\lim _{x \rightarrow 2} \frac{x^{2}+x-6}{x-2}$
(b) $\lim _{x \rightarrow 3}\left(2 x^{2}-x+\sqrt{3 x}\right)$
(d) $\lim _{x \rightarrow 1} \frac{|4 x-4|^{2}}{|4 x-4|}$
(2) Determine if the following limits exist, and evaluate them if they do.
(a) $\lim _{x \rightarrow 0} \frac{e^{x}+1}{e^{2 x}-1}$
(c) $\lim _{x \rightarrow 0} \cos (x) \frac{|x|}{x}$
(b) $\lim _{x \rightarrow 0} x \sin (3 x)$
(!) (d) $\lim _{x \rightarrow 0} x^{2} e^{\sin \left(\frac{1}{x}\right)}$
(3) For each of the following, find a value of $k$ so that the limit exists.
(a) $\lim _{x \rightarrow 1} \frac{x^{2}-k}{x-1}$
(b) $\lim _{x \rightarrow 0}\left(\frac{k}{x}-\frac{4}{k x}\right)$
(c) $\lim _{x \rightarrow 1} f(x)$ where $f(x)= \begin{cases}2 k x & x<1 \\ x^{2}+k & x \geq 1\end{cases}$
(4) Imagine that a rope is stretched around the equator of the earth (for the sake of simplicity, assume that the earth is a perfect sphere). Now, extend the rope by 20 feet, while still keeping it held in a circular shape centered on the earth. With only the information given, is it possible to figure out how high off the ground the new rope is? If so, how high will the rope be?
(!!) (5) In 180, it will be sufficient for us to work with an intuitive understanding of what a limit is. However, the definition we work with is a little imprecise (what exactly does it mean for $f(x)$ to get arbitrarily close to $L$ ? Or $x$ close to $a$ ? Beats me).

To be a little more precise, here's the "real" definition of a limit (if you ever take Math 313, you'll work a lot with this):

Definition. The limit $L=\lim _{x \rightarrow a} f(x)$ exists if for any positive number $\epsilon$, there is another positive number $\delta$ such that whenever $x$ is a real number and $|x-a|<\delta$, then $|f(x)-L|<\epsilon$.
(a) In plain English, what is this definition saying and how does it relate to the 180 definition of a limit?
(b) For each limit, and given $\epsilon$, find a $\delta$ that works:
(i) $\lim _{x \rightarrow 0} x$ and $\epsilon=1$
(iv) $\lim _{x \rightarrow 0} x$ and $\epsilon=1 / 2$
(ii) $\lim _{x \rightarrow 0} 3 x$ and $\epsilon=1 / 2$
(v) $\lim _{x \rightarrow 5} 3 x$ and $\epsilon=1$
(iii) $\lim _{x \rightarrow 0} x^{2}$ and $\epsilon=1$
(vi) $\lim _{x \rightarrow 1} \frac{1}{x}$ and $\epsilon=1$
(!!!) (c) For the limits in (i), (ii), (iii), given any value of $\epsilon>0$, can you figure out what the value of $\delta$ needs to be? Your answer should be in terms of $\epsilon$.

