ESP Math 179 | # Worksheet 3 – More Limits | Spring 2020

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(1) **Warm-up:** Let $f(x)$ be a function with $f(1) = 10$ and $f(2) = -2$, and $h(x) = f(x+1)$. Evaluate the following expressions:

(a) $h(0)$

(b) $h(1)/2$

(c) $f\left(\sqrt{\ln(e^4)}\right)$

(d) $f(h(f(2)+3)+3)+3$

(2) Are the following statements true or false? If true, provide a brief explanation of why, and if false, give an example that shows the statement failing.

(a) If $\lim_{x\to a} f(x) = L$, then $f(a) = L$.

(b) If $\lim_{x\to a^+} f(x) = L$, then $\lim_{x\to a^-} f(x) = L$.

(c) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = L$, then $f(a) = g(a)$.

(d) If $g(a) = 0$, then the limit $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ does not exist.

(e) If $\lim_{x\to 1^+} \sqrt{f(x)} = \sqrt{\lim_{x\to 1^+} f(x)}$, then $\lim_{x\to 1^-} \sqrt{f(x)} = \sqrt{\lim_{x\to 1^-} f(x)}$.

(3) Determine if the following limits exist, and evaluate them if they do.

(a) $\lim_{x \to 0} (\ln(x+1))$

(b) $\lim_{x \to 1} \left(\frac{x^2 - 1}{x - 1} \right)$

(c) $\lim_{y \to 4} \left(\frac{y^2 - y - 12}{\sqrt{y} - 2} \right)$

(d) $\lim_{z \to 3} \left(\frac{3 - z}{z - 3} \right)$

(e) $\lim_{w \to 3} \left(\frac{|w-3| + |w-3|}{w - 3} \right)$

(f) $\lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{2}{x^2 - 2x} \right)$

(4) Give an example of two functions $f(x)$ and $g(x)$ such that $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ do not exist, but $\lim_{x \to 0} (f(x) + g(x))$ does exist.

(5) (a) Suppose $f(x) = 1$ when x is an irrational number and $f(x) = 0$ when x is a rational number. Does $\lim_{x \to 0} f(x)$ exist, and if so, what is its value?

(b) Suppose $g(x) = 0$ when x is an irrational number and $g(x) = x^2$ when x is a rational number. Does $\lim_{x \to 0} f(x)$ exist, and if so, what is its value?