ESP Math 179 | Spring 2020

Worksheet 5

Sequences & Infinite Limits

28 January 2020

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1. Sequences

Sequences aren't covered until Calc 2, but they can be helpful in providing another way of thinking about limits. A **sequence** is an ordered, unending collection of numbers, where the numbers are allowed to repeat. We will often label the elements of a sequence using variables with subscripts like $x_1, x_2, x_3, \ldots$.

Definition. Given a sequence $x_1, x_2, \ldots$, L is the **limit of the sequence as n goes to infinity**, written $L = \lim_{n\to\infty} x_n$, if the values of x_n become arbitrarily close to L as n gets large.

(1) Find a formula for the n^{th} term of the following sequences. The first term given is for $n = 1$.

(a) $1, 4, 9, 16, 25, \ldots$

(b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

(c) $2, 4, 8, 16, 32, 64, \ldots$

(!) (d) $\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \frac{3}{32}, \frac{7}{128}$

(2) For each sequence, determine if the limit exists as n goes to infinity, and if it does, find the limit.

(a) $x_n = \dfrac{1}{n}$

(b) $y_n = \dfrac{n^3}{n^4 + 1}$

(c) $z_n = \dfrac{n^3 - n}{n^2 + 2n + 1}$

(d) $3, 3.1, 3.14, 3.141, 3.1415, \ldots$

(e) $a_n = (-1)^n$

(f) $b_n = \dfrac{(-1)^n}{n}$

(g) $c_n = \cos(n\pi)$

2. Infinite Limits

(3) Fill in the blanks.

(a) A function can have at most _____ horizontal asymptotes.

(b) A function can have at most _____ vertical asymptotes.

(4) Find the following limits, if they exist:

(a) $\lim_{x\to\infty} \dfrac{3x^5 - 2x}{x^5 + 3x^4}$

(b) $\lim_{x\to-\infty} \dfrac{x^4 + 2x^2 - 1}{x^5 - 2}$

(c) $\lim_{x\to\infty} \dfrac{x^3 - 2x^2}{x^2 + 3x - 1}$

(!) (d) $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$ (try plugging in large positive values for x)

(!) (e) $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$ (try plugging in large negative values for x)

(5) Find all vertical asymptotes $x = a$ of the function

$$f(x) = \frac{(x-2)(x+2)}{(x+3)(x-1)^2}.$$

For each value of a, find $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, and $\lim_{x\to a} f(x)$.

(!) (6) Recall that a rational function is a ratio of two polynomial functions:

$$f(x) = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_2 x^2 + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_2 x^2 + b_1 x + b_0}$$

Prove that if $m = n$, then $\lim_{x\to\pm\infty} = \frac{a_m}{b_n}$.