ESP Math 179

Worksheet 8

Computing Derivatives

6 February 2020

Spring 2020

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(1) (**Warm-up**) Are the following true or false?

  (a) If a line is tangent to a graph at a point, it only touches the graph at that one point.

  (b) The exponential function e^x has two different points with the same tangent lines.

  (c) For every function $f(x)$, after taking 'enough' derivatives of $f(x)$ we get zero. (As an example, the derivative of $2x$ is 2, and the derivative of 2 is 0)

(2) Evaluate the derivatives of the following functions.

  (a) x^5

  (b) $3x^2$

  (c) e^x

  (d) x^{-1}

  (e) $3x^2 + 2x$

  (f) $(x^5)(3x^2)$

  (g) $\dfrac{x^5}{2x+1}$

  (h) $(x^{-1})(e^2 x)$

(3) Using the limit definition of the derivative, show that the derivative of the sum of two functions is the sum of the derivatives, i.e. show that for any differentiable f, g, $(f+g)' = f' + g'$.

(4) (a) Using the product rule, prove that $\dfrac{d}{dx}x^2 = 2x$.

  (b) Using the product rule, prove that $\dfrac{d}{dx}x^3 = 3x^2$.

  (c) Using the product rule, prove that $\dfrac{d}{dx}x^4 = 4x^3$.

  (d) Do you see how this argument can generalize to prove the power rule?

(5) For this problem, we will need the idea of **higher-order derivatives**. Recall that $\frac{d}{dx}f(x)$ is the derivative of $f(x)$ with respect to x. The **n-th derivative** of $f(x)$ with respect to x, denoted $\frac{d^n}{dx^n}f(x)$, is the result of taking the derivative n times. For example,

$$\frac{d^2}{dx^2}x^3 = \frac{d}{dx}\left(\frac{d}{dx}x^3\right) = \frac{d}{dx}3x^2 = 6x$$

Evaluate the following derivatives.

(a) $\frac{d}{dx}x$

(b) $\frac{d^2}{dx^2}x^2$

(c) $\frac{d^3}{dx^3}x^3$

(d) $\frac{d^4}{dx^4}x^4$

(e) Can you guess the value of $\frac{d^n}{dx^n}x^n$, where n is a natural number?

(**!**) (6) Use the product rule to prove the quotient rule.

(**!!**) (7) Use the limit definition of the derivative to prove the product rule.