ESP Math 179

Worksheet 7

Derivatives

4 February 2020

Spring 2020

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(1) Give an example of a function that is

(a) not continuous and not differentiable at $x = 0$

(b) continuous and not differentiable at $x = 0$

(c) continuous and differentiable at $x = 0$

(2) True or false? Explain your answer.

(a) If a function has a derivative, then the derivative is continuous.

(b) Two different functions can have the same derivative.

(c) A function can be equal to its own derivative.

(3) Using the definition of a derivative, find the derivatives of the following functions.

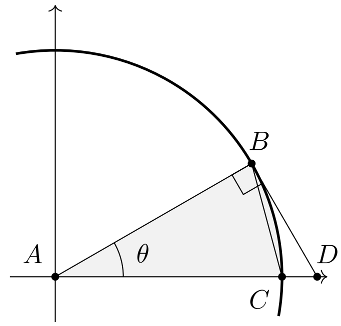
(a) $f(x) = 2x^2 + 1$

(b) $f(x) = \dfrac{1}{x}$

(c) $f(x) = x^3 + x^2$

(d) $f(x) = e^x$ (you may use the fact that $\displaystyle\lim_{h\to 0} \frac{e^h - 1}{h} = 1$)

(4) Consider the unit circle (circle of radius 1) in the first quadrant, as below.



(a) In terms of $\sin(\theta)$ and $\cos(\theta)$, express the area of the triangle ABC.

(b) In terms of $\sin(\theta)$ and $\cos(\theta)$, express the area of the triangle ABD. (Hint: use the law of sines)

(c) The area of the sector (shaded area) ABC is $\frac{1}{2}\theta$. It should also be easy to observe that

(area of triangle ABC) $\leq$ (area of sector ABC) $\leq$ (area of triangle ABD),

Given these two facts, prove that $1 \leq \dfrac{\theta}{\sin(\theta)} \leq \dfrac{1}{\cos(\theta)}$.

(d) Use the squeeze theorem to evaluate $\lim\limits_{\theta \to 0} \dfrac{\sin(\theta)}{\theta}$.

(e) Use the trig identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to show $\lim\limits_{\theta \to 0} \dfrac{1 - \cos(\theta)}{\theta} = 0$

(f) Use the trig identities $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$ and $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ to find the derivatives of $\sin(x)$ and $\cos(x)$.

(!!) (5) Prove that if f is differentiable at $x = a$, then f is continuous at $x = a$.