ESP Math 179

Worksheet 10

Chain Rule

Spring 2020

12 February 2020

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1. (**Warm-up**) Compute the following limits, or state that they do not exist.

   (a) $\displaystyle\lim_{x\to 3} \frac{x^2 - x - 6}{x - 3}$

   (b) $\displaystyle\lim_{x\to 5^+} \frac{2x - 3}{x - 5}$

   (c) $\displaystyle\lim_{x\to \infty} \frac{x^2 + x - 3}{3x^3 + x^2 - 10x + 7}$

   (d) $x^4 \sin\left(\dfrac{1}{x^4}\right)$

2. (**Warm-up**) Find the derivatives of the following functions. Do not simplify.

   (a) $f(x) = x^2 + \sqrt{x}$

   (b) $g(x) = 3x^2 + 2x - 1$

   (c) $h(x) = e^{2x}\sin(x)$

   (d) $s(x) = \dfrac{\cos(x)}{x^2 + 2x}$

3. Use the chain rule to calculate the derivatives of the following functions. Do not simplify!

   (a) $(2x^2 - 4x + 2)^6$

   (b) $(\sin(x))^3$

   (c) $\tan(\sqrt{x})$

   (d) e^{-x^2}

   (e) $\cos(x^3 + x^2)$

   (f) $((x^2 + 3)^3 + x^2 - 1)^2$

   (g) $(\sin((2x + 1)^3))^4$

4. (a) Use the fact that $e^{\ln(x)} = x$, along with the chain rule, to compute $\frac{d}{dx}\ln(x)$.

(b) Let a be a positive number. Compute $\frac{d}{dx}a^x$. (hint: rewrite a^x as $e^{a\ln(x)}$)

(**!**) (c) Compute $\frac{d}{dx}x^x$

(c) Recall that an inverse function for f is another function g, such that $f(g(x)) = x$. Use this to show that

$$g'(x) = \frac{1}{f'(g(x))}$$

How could you use this to do part (a)?

5. Use the product and chain rules to derive the quotient rule. That is, given two arbitrary differentiable functions f and g, using only the chain and product rules, compute $\frac{d}{dx}\frac{f(x)}{g(x)}$.

6. Compute the derivative of $e^{e^{e^{e^x}}}$.