ESP Math 179 | Spring 2020

Worksheet 9

Trig Functions

11 February 2020

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(1) The midterm is in exactly one week! On another sheet of paper, make a short plan/outline of how you're going to study – include both when you're going to study and what you're going to do during that time (i.e., review notes, watch study videos, work on past discussion/homework problems, work on the exam review problems, etc.).

(2) Use the quotient rule to evaluate the following derivatives. Recall that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, $\csc(x) = \dfrac{1}{\sin(x)}$, $\sec(x) = \dfrac{1}{\cos(x)}$, $\cot(x) = \dfrac{1}{\tan(x)}$:

(a) $\dfrac{d}{dx}\tan(x)$

(c) $\dfrac{d}{dx}\sec(x)$

(b) $\dfrac{d}{dx}\csc(x)$

(d) $\dfrac{d}{dx}\cot(x)$

(3) Evaluate the following derivatives:

(a) $\dfrac{d}{dx}x^2\sin(x)$

(b) $\dfrac{d}{dx}e^{2x}\csc(x)$

(c) $\dfrac{d}{dx}\sin(x)\tan(x)$

(d) $\dfrac{d}{dx}\dfrac{\sin(x)}{1+\tan(x)}$

(e) $\dfrac{d}{dx}\dfrac{\cos(x)}{3x^2+\sin(x)}$

(f) $\dfrac{d}{dx}\dfrac{\sin(x)\cos(x)}{2x+\cot(x)}$

(4) Recall the factorial function $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$. Calculate the following derivatives. Do you see a pattern?

(a) $f_0(x) = 1$

(b) $f_1(x) = 1 + x$

(c) $f_2(x) = 1 + x + \frac{x^2}{2!}$

(d) $f_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

(e) $f_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

(f) $f_5(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$

(5) Calculate the derivative of the following function in terms of n:

$$f_n(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$$

(6) For the function $f_n(x)$, we can get $f_{n+1}(x)$ by adding $\frac{x^{n+1}}{(n+1)!}$. Imagine that we kept adding on more terms an infinite number of times, to get a function

$$f_\infty(x) = 1 + \frac{x}{1!} + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!} + \frac{x^{n+2}}{(n+2)!} + \cdots$$

What is $f'_\infty(x)$? (Technically, we need to be a little careful because there are an infinite number of terms. For now, you can assume that $f'_\infty(x)$ is the sum of the derivatives of each term).

(7) Based on your result for question (5), can you guess what the function f_∞ is? It should be something that we are familiar with and know the derivative for.