ESP Math 179 | **WORKSHEET 11** | Spring 2020

EXAM 1 REVIEW

18 February 2020

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1. For each of the following problems, a sample solution is given. Identify whether or not the solution is correct. If incorrect, identify what the issue is.

   (a) Find the derivative of $e^{\cos(x)}$.

   Solution: $\left(e^{\cos(x)}\right)' = e^{-\sin(x)}$, since $(e^x)' = e^x$ and $(\cos(x))' = -\sin(x)$.

   (b) Find any vertical asymptotes of the function $f(x) = \dfrac{x^2-4x+4}{x^2-4}$, and justify using limits.

   Solution: We factor and simplify $f(x)$:

$$f(x) = \frac{x^2-4x+4}{x^2-4} = \frac{(x-2)^2}{(x-2)(x+2)} = \frac{x-2}{x+2},$$

   which tells us that $x=-2$ is a vertical asymptote. To see this using limits, we see that

$$\lim_{x\to -2^-} \frac{x-2}{x+2} = -\infty,$$

   since plugging in $x=-2$ to the simplified form of $f(x)$ gives us $\frac{4}{0}$, and when x is a little less than -2, $x+2$ is negative. So we get positive divided by negative which is negative. Since one of the one-sided limits is $\pm\infty$, this is a vertical asymptote.

   (c) Find the equation of the tangent line to $f(x) = \cos(2x)$ at $x=\pi$.

   Solution: To find the tangent line, we need the slope and a point on the line. To find the slope, we find $f'(x)$. This is

$$(\cos(2x))' = -2\sin(2x)$$

   by using the chain rule. We then plug in $x=\pi$ to find the slope of $f(x)$ when $x=\pi$. This is $-2\sin(2\pi) = 0$.

   Next, we find a point on the tangent line. We can do this by plugging in $x=\pi$ into $f(x)$, which is $f(\pi) = \cos(2\pi) = 1$. This gives us the point $(\pi, 1)$.

   So the equation of the tangent line is $y-1 = 0(x-\pi)$ using point-slope form, which then simplifies to $y=1$.

   (d) Use the limit definition of the derivative to find the derivative of $f(x) = x^2$.

   Solution: The limit definition of the derivative is

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

   For x^2, this gives us

$$\begin{aligned}\left(x^2\right)' &= \lim_{h\to 0} \frac{x^2+h-x^2}{h}\\ &= \lim_{h\to 0} \frac{h}{h}\\ &= \lim_{h\to 0} 1\\ &= 1\end{aligned}$$

   So the derivative of x^2 is 1.

(e) Evaluate the following limit. If the limit does not exist, explain why.

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$

Solution: We first try plugging in $x = 0$. This gives us

$$0^2 \cos\left(\frac{1}{0}\right)$$

which is undefined since $\frac{1}{0}$ is undefined. Therefore, the limit does not exist.

2. Evaluate the following limits. If the limit does not exist, explain why.

   (a) $\displaystyle\lim_{x \to 2} \frac{x-3}{(x-2)(x-5)}$

   (b) $\displaystyle\lim_{y \to 5} \frac{y-5}{y^2-7y+10}$

   (c) $\displaystyle\lim_{z \to 3} \frac{z-3}{|z-3|}$

3. Find the derivatives of the following functions. Do not simplify.

   (a) $f(x) = x^3 + 4x^2 - 1$

   (b) $g(x) = x^2 \sin(x)$

   (c) $h(x) = \dfrac{\tan(x)}{x^3 + x^2}$

   (d) $r(x) = \sin(e^x)$

   (e) $s(x) = x^3 \tan(x) e^x$

   (f) $t(x) = e^{\cos(x^2)}$

4. Where is the following function continuous? Where is it differentiable? (try sketching the graph)

$$f(x) = \begin{cases} -2 & x < -1 \\ x - 1 & -1 \leq x < 1 \\ x^2 - 4x + 4 & 1 \leq x < 3 \\ 1 & 3 \leq x < 2\pi \\ \cos(x) & x \geq 2\pi \end{cases}$$

5. Suppose you go on a hike somewhere outside Chicago, where hills actually exist. Assume that the start of the trail is neither the lowest nor the highest point of the trail. Show that if you make a round trip on your hike, you must reach some other point with the same altitude as the start of the trail. (hint: use the Intermediate Value Theorem)