ESP Math 179 Spring 2020

Worksheet 12

Implicit Differentiation
Logarithmic Differentiation
Inverse Trig Functions

25 February 2020

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1. The first midterm grades are up on Gradescope. If you haven't already, take a look at your exam. What went well? What didn't go well? What topics will you need to spend some time reviewing more? What might you want to do differently to prepare for upcoming exams? Use the space below to write down your answers to these questions, and feel free to write down any other reflections about the exam.

2. (**Warm-up**) Using limits, find any horizontal asymptotes of the function $f(x) = \dfrac{3x^2 + 2x - 1}{x^2 + 210x - 9965}$.

3. (**Warm-up**) Compute the derivative of $g(x) = \dfrac{3x^2 e^x}{x+1}$.

4. For the following equations, find y' by implicit differentiation, then use that to find the tangent line at the given point.
   (a) $x^4 + y^2 = 3$ at the point $(1, -\sqrt{2})$.

   (b) $y = 2x^2 y - y^3$ at the point $(1, 1)$.

   (c) $y^2 e^{2x} = 3y + x^2$ at the point $(0, 3)$.

5. Use logarithmic differentiation to find the derivatives of the following functions.
   (a) x^x

   (b) $(\cos x)^{x+1}$

   (c) $x^{\ln x}$

6. Evaluate the derivatives of the following functions.
   (a) $\sin^{-1}(3x^2)$

   (b) $\cos^{-1}(e^x)$

   (c) $\tan^{-1}(x + \sin x)$

7. In this problem, we will compute the derivative $\frac{d}{dx}\sin^{-1}(x)$ from scratch.

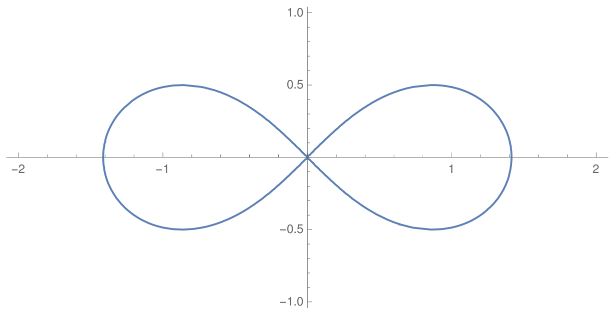
   (a) Recall that $y = \sin^{-1}(x)$ means that $\sin(y) = x$. Using implicit differentiation, find an expression for $\frac{dy}{dx}$ (your expression will include y).

   (b) Your expression above should have $\cos(y)$ in it. However, we don't know what $\cos(y)$, but we do know what $\sin(y)$ is. Use the equation $\sin^2(y)+\cos^2(y) = 1$ to rewrite your above expression so that it uses $\sin(y)$.

   (c) Use what we know about $\sin(y)$ to rewrite the previous expression in terms of x (no y's should appear). This should give you $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$.

8. Repeat what you did above (with the appropriate modifications) to compute the derivative $\frac{d}{dx}\cos^{-1}(x)$ from scratch.

9. Below is the graph of $(x^2 + y^2)^2 = 2x^2 - 2y^2$.



Just by looking at the graph, answer the following questions.

(a) How many points on the graph are there for which $y' = 0$?

(b) Fix some real number c. How many points on the graph are there for which $y' = c$?

10. We know that for any constant C, the derivative of $f(x) = Ce^x$ is the same function, i.e. $f'(x) = Ce^x$. This might raise the question: are there any other functions with this property? To explore this, consider the following:

(a) Assume $f(x) = f'(x)$. Define a new function $g(x) = \dfrac{f(x)}{e^x}$. Find $g'(x)$. Make sure to simplify!

(b) What does this tell you about $g(x)$? From this, what can you say about $f(x)$?