ESP Math 179

# Worksheet 14

## Extrema & the Mean Value Theorem

3 March 2020

Spring 2020

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1. (**Review**) Compute the derivative of $(\sin x)^{2x}$. Do not simplify.

2. (**Review**) A 10-foot ladder is leaning against a wall. It begins to slip down the wall, with the point touching the wall dropping at a rate of 1 foot/sec. At the moment that the top of the ladder is 8 feet above the ground, how quickly is the bottom of the ladder moving away from the wall?

3. (**Warm-up**) For a function $f(x)$, define critical points, local maximum/minimum, and global maximum/minimum.

4. (**Warm-up**) What is the process for finding the absolute maximum and minimum?

5. (**Warm-up**) In words, what does the Mean Value Theorem say?

6. Determine the absolute maximum and minimum for $f(x) = x^3$ on the interval $[-3, 5]$.

7. Determine the absolute maximum and minimum for $f(x) = 8x^3 + 81x - 42x - 8$ on the interval $[-8, 2]$.

8. Suppose that we know that $f(x)$ is continuous and differentiable on the interval $[-7, 0]$, that $f(-7) = -3$, and that $f'(x) \leq 2$ for all $x$ in $[-7, 0]$. What is the largest possible value for $f(0)$?

9. Recall that any quadratic function can be written in the form $f(x) = ax^2 + bx + c$, where $a, b, c$ are real number constants.

   (a) What does it mean for two real numbers $r$ and $s$ to be roots (or zeros) of $f(x)$?

   (b) If $r$ and $s$ are roots of $f(x)$, then we can factor to write $f(x)$ in the form $f(x) = k(x-r)(x-s)$, where $k$ is a real number constant. Sketch an example of a quadratic function and label the roots $r$ and $s$.

   (c) Use the equation from part (b) to show that $f'(r) = -f'(s)$. What does this mean geometrically? (look at your graph)

   (d) Show that $f(x)$ has a critical point halfway between $r$ and $s$. Again, what does this mean geometrically? (hint: the halfway point between $r$ and $s$ is $\frac{r+s}{2}$)

(**!**) 10. Recall that **Rolle's Theorem** tells us that if $f$ is continuous on a closed interval $[a, b]$ and differentiable on the open interval $(a, b)$, with $f(a) = f(b)$, then there is some point $c \in (a, b)$ such that $f'(c) = 0$.

This statement looks eerily similar to the Mean Value Theorem. Use Rolle's Theorem to prove the Mean Value Theorem.