ESP Math 179

Worksheet 17

Optimization & Review

Spring 2020

28 January 2020

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1. (**Warm-up:**) In your own words, what is the goal of an optimization problem?

2. (**Warm-up:**) What are the steps to solving an optimization problem?

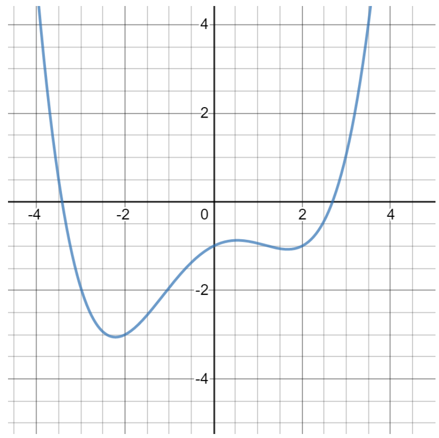
3. A rancher has 2400 ft of fence available to enclose a rectangular field that borders a straight river. If she doesn't need any fencing along the river, what is the largest area of field she can enclose (and what will be the dimensions for the field)?

4. Suppose that the cost of manufacturing an aluminum soda can only depends on the surface area of the can (assume that a can is a perfect cylinder). Find the dimensions of a can that holds 12 ounces of liquid using the least amount of aluminum.

   Now suppose it costs twice as much to make the top and bottom of the can compared to the side. What dimensions minimize the cost in this case?

5. Find the equation of the tangent line to the curve $x^2 + y^3 + xy = 1$ at the point $(0, 1)$.

6. Consider the graph of a function below:



For the following, you can give approximate answers.

(a) What are the intervals of increase and decrease?

(b) What are the intervals of concavity?

(c) Where are the critical points? For each critical point, is it a local max, min, or neither?

7. Apparent brightness is related to distance by the *inverse square law*, which basically says that as you move away from an object, the brightness decreases by a factor of the square of the distance you move. Let's be a bit more precise: *lux* is a unit for measuring brightness, and an *au* (astronomical unit) is the distance between the Earth and the Sun (about 150 million km). If the sun appears to be C lux from the Earth, then to an observer r au away, the sun will appear to be $\frac{C}{r^2}$ lux.

Suppose a spaceship is travelling away from the Sun at a speed of 2 au per year. When the ship is 3 au away from the Sun, what is the rate of change of the apparent brightness of the sun?

8. Max was driving along the tolled part of I-90. He reached one toll plaza at 1:02pm, and arrived at another toll plaza 60 miles away at 1:58pm. Based on this, he received a ticket for speeding (the speed limit is 60mph the entire way). How can the court justify this ticket? You should be citing a theorem we have learned in calculus.