Propositional Logic

Defn A **proposition** is a statement that can either be true or false.

A **propositional variable** is a symbol that represents a proposition.
Propositional variables may be assigned a truth value of "true" or "false".

eg p represents "the sky is cloudy today."

propositional variable   proposition

Defn A **propositional statement** (or propositional formula) is an expression that either is a propositional variable, or is constructed from simpler propositional statements and logical connectives.

Logical connectives

1) Conjunctions ("and", $\wedge$)

- if p is true and q is true, then $p \wedge q$ is true
- if $p \wedge q$ is true, then p and q are both individually true.

- to prove "$p \wedge q$", give a proof of p and a proof of q.
- when assuming "$p \wedge q$" in a proof, we may assume p and q individually in the proof.

2) Disjunctions ("or", $\vee$)

- if p is true, then $p \vee q$ is true
- if q is true, then $p \vee q$ is true.
- if $p \vee q$ is true, then at least one of p and q is true.

- to prove "$p \vee q$", we only need to prove one of the two.
- when assuming "$p \vee q$" in a proof, we should split into two cases:
  - one where we assume p is true, and
  - one where we assume q is true.

3) Negations ("not", $\neg$)

- "$\neg p$ is true" means that p is false, and
- "$\neg p$ is false" means that p is true.
- to prove "$\neg p$", prove that p is false.
  (we'll discuss this more in a later session)
- when assuming "$\neg p$" in a proof, we can assume that p is false.

4) Implications ("If ... then ...", $\Rightarrow$, $\rightarrow$)

- If, under the assumption that p is true, then q must also be true, then "$p \Rightarrow q$" is true.
- if "$p \Rightarrow q$" is true, and we know p is true, then q is also true.

- to prove "$p \Rightarrow q$", first assume that p is true, then use this to derive that q is true.
- when assuming "$p \Rightarrow q$" in a proof, then if p is assumed to be true, we may also assume that q is true.

Note "$p \Rightarrow q$" has the same truth value as "$(\neg p) \vee q$".

5) Biconditionals ("if and only if", "iff", $\Leftrightarrow$, $\leftrightarrow$)

- "$p \Leftrightarrow q$" means "$(p \Rightarrow q) \wedge (q \Rightarrow p)$".

Truth Tables

p	$\neg p$
T	F
F	T

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

eg

p	q	r	$(p \wedge q) \vee (p \wedge r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Logic Puzzles

On a remote island, everyone is either a truth-teller or a liar.

John says, "Bill and I are both liars."

Case analysis

Truth table