Week 1

INTRODUCTION & WHAT IS A PROOF?

January 11, 2021

- 0. Introductions: in your groups, discuss the following questions.
 - (a) What has been your experience with math so far? This can include math classes you've taken, math that you've needed for other subjects, or extracurricular experiences.
 - (b) How do you feel about math currently (e.g., do you enjoy it? dislike it?)? Why do you think you feel that way?
 - (c) Based on the goals for this workshop that were discussed earlier, what sort of expectations do you have for the course? What do you hope to get out of this class?

Note: since this is the first week, I absolutely don't expect you to necessarily solve the problem or have a well-polished write-up. The problems for this week are mainly to give you a flavor of what is to come, and to give you a sense of how difficult effective proof-writing can be. As we go through the semester, we'll learn some good techniques for solving these sorts of problems. By the end of the semester, these problems should become a breeze!

- 1. Prove that the sum of two even numbers is an even number, and that the sum of two odd numbers is an even number.
- 2. Prove that if n is even, then n^2 is even, and if n is odd, then n^2 is odd.
- 3. If a and b are integers, we say that a divides b if there is an integer r so that b = ar (think of this as saying that $\frac{b}{a}$ gives you a whole number). Prove that if a divides b and b divides c, then a divides c.
- 4. Suppose that a < b. Prove that $a < \frac{a+b}{2} < b$. (Hint: you should prove that $a < \frac{a+b}{2}$ and that $\frac{a+b}{2} < b$ are true separately.)
- 5. (Challenge) Prove that $\sqrt{2}$ is an irrational number (a rational number is a number that can be written as a fraction $\frac{p}{q}$, where p and q are both integers, while an *irrational number* is a number that, well, isn't rational).