Week 11

Injections, Surjections, & Bijections

March 28, 2021

---

1. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = 2n + 1$. Is f injective? Is f surjective? Prove your answers.

   Given a set A, we can define a function $\mathrm{id}_A : A \to A$ by $\mathrm{id}_A(a) = a$ for every $a \in A$. The function id_A is called the *identity function*, and is the function that does nothing to its inputs.

2. Let A, B be nonempty sets and $f : A \to B$ be an injective function. Prove that there is a surjection $g : B \to A$ such that $g \circ f = \mathrm{id}_A$. ($g$ is called a *left inverse.*)
3. Let A, B be nonempty sets and $f : A \to B$ be a surjective function. Prove that there is an injection $g : B \to A$ such that $f \circ g = \mathrm{id}_B$. ($g$ is called a *right inverse.*)

4. Let A be any set, and $f : A \to A$. Prove that if f is injective but not surjective, then A must have infinitely many elements.