ESP Math 294 | Spring 2021

Week 13
Combinatorics

April 13, 2021

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1. Prove Pascal's identity, which is that for all n and k, $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$.
2. Prove that for all n, k, l, $\binom{n}{k}\binom{k}{l} = \binom{n}{l}\binom{n-l}{k-l}$.
3. (a) Let X be a finite set with n elements. How many possible functions are there from X to $\{0, 1\}$? (Side note: these sorts of functions are often thought of as assigning a "yes/no" answer to each element of X.)

   (b) What other quantity have we seen that is counted by this same number? How are these two thing related?
4. Let X be a finite set with n elements and Y be a finite set with m elements. How many possible functions are there from X to Y?
5. Prove that for all $l \leq n$,
$$\sum_{k=l}^{n} \binom{n}{l}\binom{k}{l} = 2^{n-l}\binom{n}{m}$$
6. Let X be a finite set with n elements. Given an element $x \in X$ and a function $f : X \to X$, we say that f *fixes* x if $f(x) = x$ (i.e., f doesn't change x at all). Prove that the number of functions $f : X \to X$ that **exactly** k elements is $\binom{n}{k}(n-1)^{n-k}$.