ESP Math 294 | Spring 2021

Week 2

Propositional Logic & Logic Puzzles

January 17, 2021

All groups:

1. Describe the steps that should be followed in order to prove statements of the following forms (don't use a truth table):

   (a) $(p \wedge q) \Rightarrow (\neg r)$

   (b) $(p \Rightarrow q) \iff (\neg q \Rightarrow \neg p)$

2. Fontano's is an Italian sandwich shop near campus. Let p mean "you get a bag of chips and a drink for free from Fontano's", q mean "you buy a sandwich from Fontano's", r mean "you buy a slice of pizza from Fontano's", and t mean "it's a Tuesday".

   What does the following propositional statement mean in plain English?

   $$t \wedge (q \vee r) \Rightarrow p$$

   (note that this is not actually true in real life)

3. Consider the statement "The Blue Line will be delayed if it has snowed more than one inch, and if the Blue Line is delayed, Kevin will be late for his class" (we're either in pre-COVID or post-COVID times for this problem, so Kevin actually has to go to campus). Write this statement as a propositional statement. Be sure to specify what each propositional variable represents.

4. For each of the following, find a propositional statement that matches the unlabeled column in the truth table:

   (a)

p	q	
T	T	F
T	F	T
F	T	T
F	F	F

   (b)

p	q	
T	T	T
T	F	F
F	T	T
F	F	F

Group 1:

5. Zoey says, "at least one of the following is true: I am a truth-teller, or Mel is a truth-teller." Mel says, "Zoey could say that I am a liar." What is the identity of each person? Prove your answer first by a case analysis, and then by using a truth table.

6. You are given two chests. You know that inside each chest is either a treasure or a deadly trap. On chest A, there is an inscription: "At least one of these chests contains a treasure." On chest B, there is an inscription: "Chest A contains a deadly trap." You also know that either both inscriptions are true, or both are false.

   Can you choose one of the chests to open to guarantee that you will receive a treasure (and not die)? If so, which one? Prove your answer using a truth table.

Group 2:

7. Zoey says, “Mel is a liar.” Mel says, ”Zoey and I are both truth-tellers.” What is the identity of each person? Prove your answer first by a case analysis, and then by using a truth table.

8. You are given three boxes. One of them contains gold, while the other two are empty. Box 1 says, “This box is empty.” Box 2 says, “This box is empty.” Box 3 says, “The gold is in Box 2.” One of the messages is true, while the other two are false.

   Which box has the gold? Prove your answer using a truth table.

Group 3:

9. Zoey says, “of Mel and I, exactly one is a truth-teller.” Mel says, “only a liar would say that Zoey is a liar.” What is the identity of each person? Prove your answer first by a case analysis, and then by using a truth table.

10. You are lost in a maze, when you come upon three possible roads. One of them is paved with gold, the second is paved with marble, and the last is paved with stone. A guard stands in front of each road. The guard of the gold road says: “This road leads to the exit. Moreover, if the stone road leads to the exit, then so does the marble road.” The guard of the marble road says: “Neither the gold road nor the stone road lead to the exit.” The guard of the stone road says: “Follow the gold road and you’ll reach the exit; follow the marble road and you’ll be lost.”

    Which road should you take? Prove your answer using a truth table.

Challenge:

11. You’ve time-traveled back to medieval times, but the king sees you using your phone and accuses you of practicing magic, and throws you into the dungeon. However, the dungeon guard hates his job and doesn’t care if you escape or die. In the dungeon, there are two doors. One leads out of the dungeon, but the other leads to the cage where they keep hungry lions for executions (and you really don’t feel like getting eaten by lions today). The guard is willing to answer exactly one question, but you don’t know if he is a truth-teller or a liar. Can you figure out which door leads to freedom?